Physics 1901

Experimental Astronomy – Graduate Course Autumn (Apr-May 2014)

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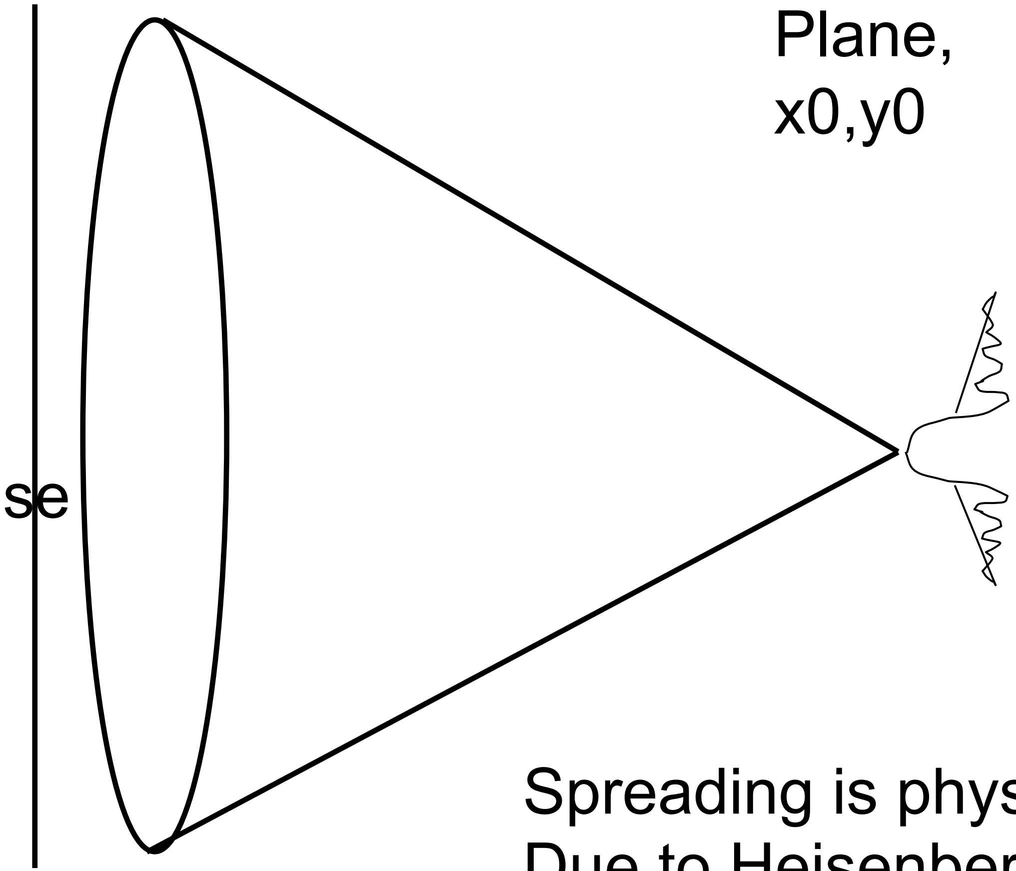
Intro to Fourier Optics

O’neill chapter 2 and 5

Goodman chapter 4,5,6



Spreading is physically
Due to Heisenberg
But how do we describe it?

Fresnel (near-field) Diffraction

Only called near field because you are in front of infinity

$$A(x_0, y_0) \propto \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\exp[\frac{\pi i}{\lambda D}(x^2+y^2)]\exp[\frac{-2\pi i}{\lambda D}(x_0 x + y_0 y)]dxdy$$

Hard, needs to be solved numerically except for all but simplest cases

$$D >> \frac{(x^2+y^2)_{\max}}{\lambda}$$

Fraunhofer (far-field) diffraction

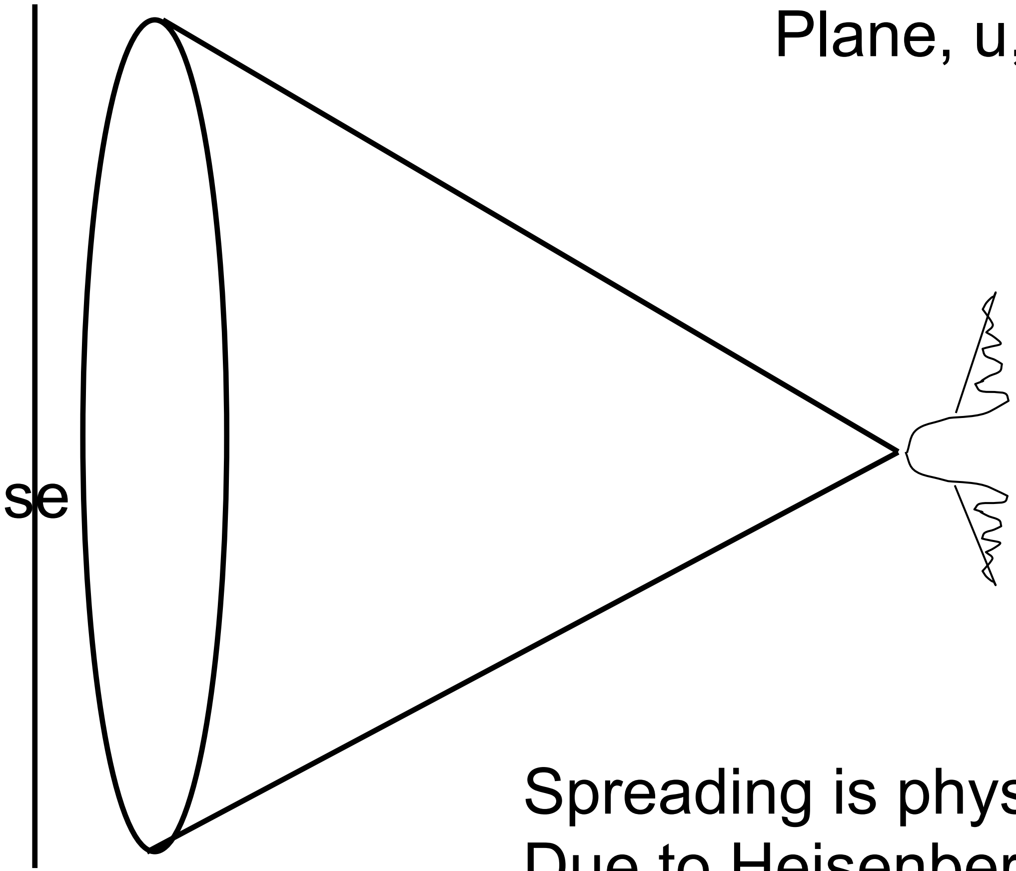
$$A(u,v) \propto \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\exp[-2\pi i(ux+vy)]dxdy = \Im\{f(x,y)\}$$

$$u = x_0/\lambda D,\ m = y_0/\lambda D,$$

Case is simplified for D= infinity, or
The back focal plane of a lens!
In these cases the E or B field is
described by a Fourier Transform

Image
Plane, u,v

Wavefront
E or B field
Amplitude and Phase



Spreading is physically
Due to Heisenberg
But how do we describe it?

Aperture
plane, x,y

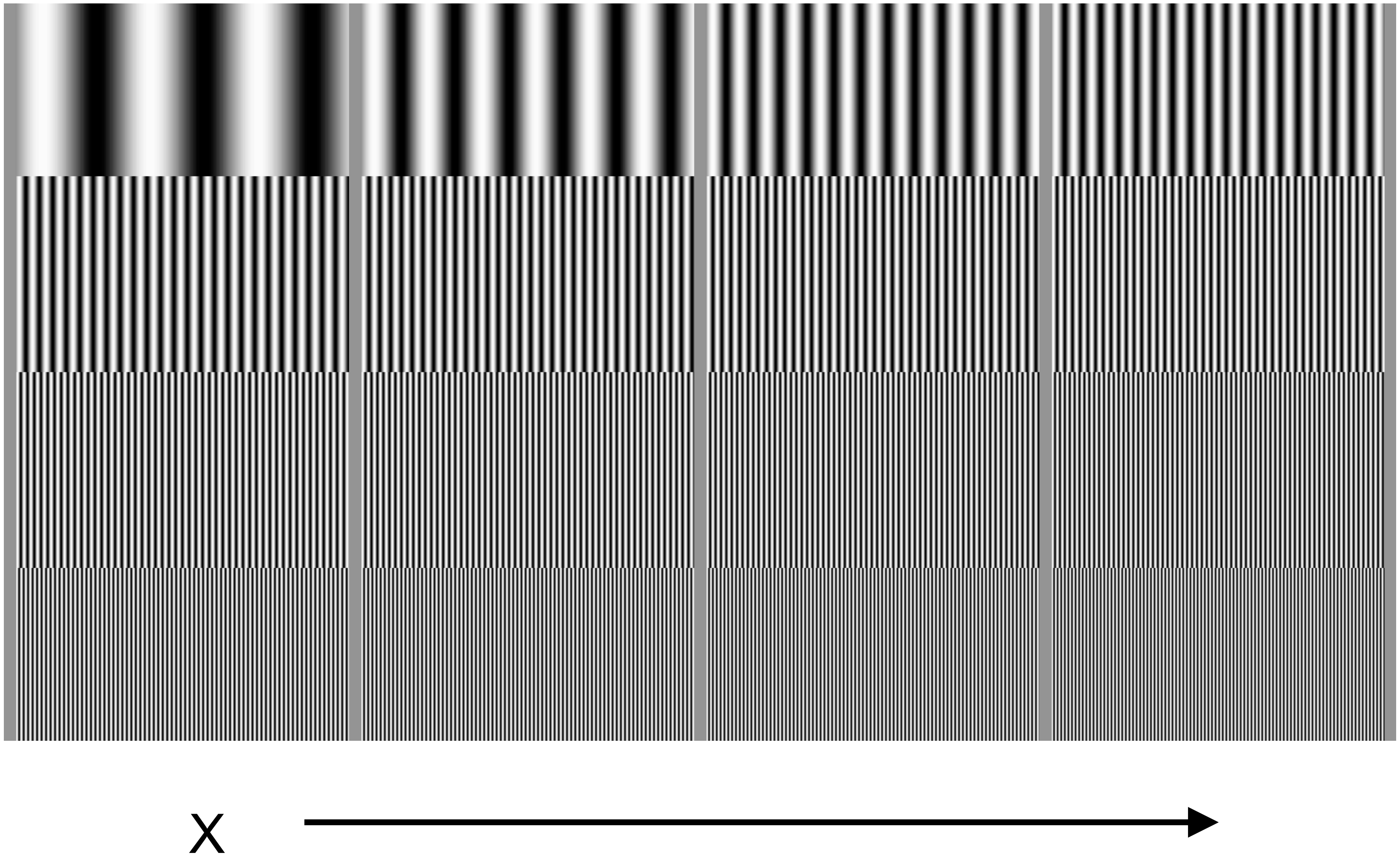
Fourier Transform (FT)

$$1D: F(u) = \Im\{f(x)\} = \int_{-\infty}^{\infty} f(x) e^{-2\pi iux} dx$$

$$2D: F(u,v) = \Im\{f(x,y)\} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x,y) e^{-2\pi i(ux+vy)} dxdy$$

Spatial Frequency

U=1/x



Inverse Transform

$$1D: f(x) = \Im^{-1}\{F(u)\} = \int_{-\infty}^{\infty} F(u) e^{2\pi i u x} du$$

$$2D: f(x, y) = \Im^{-1}\{F(u, v)\} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(u, v) e^{2\pi i (ux + vy)} du dv$$

Fourier Transform Relations

- Fourier(fourier)

$$\Im\{\Im\{f(x)\} = f(-x)$$

- Similarity Theorem

$$\Im\{f(ax)\} = \frac{1}{|a|}F\left(\frac{u}{a}\right)$$

- Addition Theorem

$$\Im\{f(x)+g(x)\} = F(u)+G(u)$$

- Shift Theorem

$$\Im\{f(x-a)\} = e^{-i2\pi as}F(u)$$

Fourier Transform Relations

- Fourier Convolution Theorem

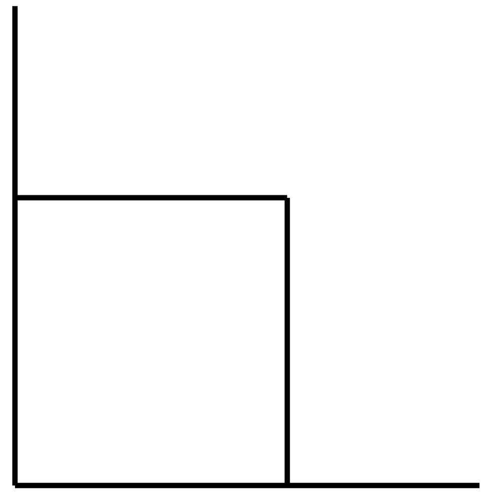
$$\Im\{\int_{-\infty}^{\infty} f(x')g(x-x')dx'\} = \Im\{f(x)\}\Im\{g(x)\}$$

- Wiener-Khintchine Theorem

$$\Im\{|F(u)|^2\} = \int_{-\infty}^{\infty} f(x')f^*(x-x')dx'$$

Useful Transforms
Tophat

$$f(x) = \Pi(x) = \begin{cases} h.........-b/2 < x < b/2 \\ 0.........otherwise \end{cases}$$



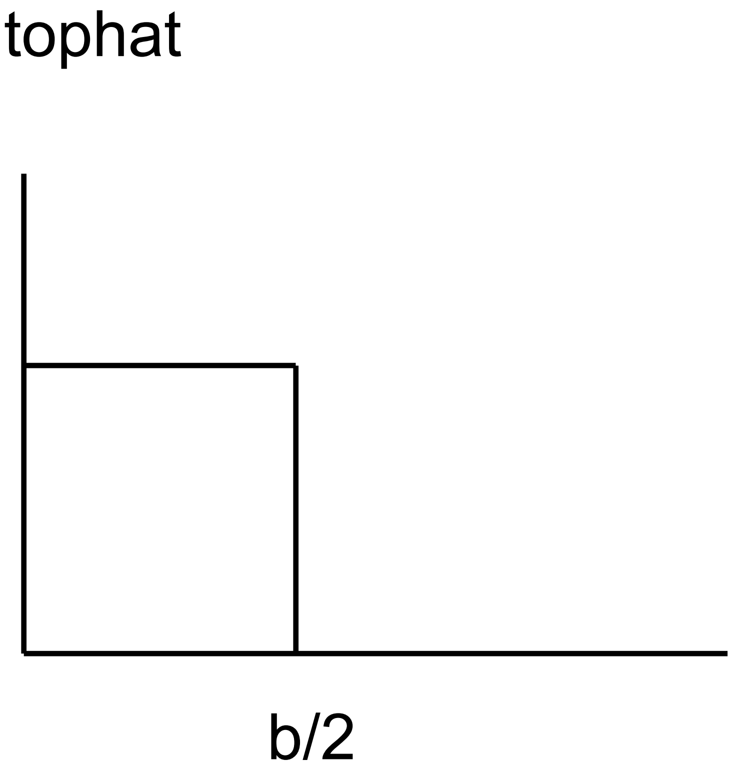
$$\mathrm{sinc}(x) = \frac{\sin \pi x}{\pi x}$$

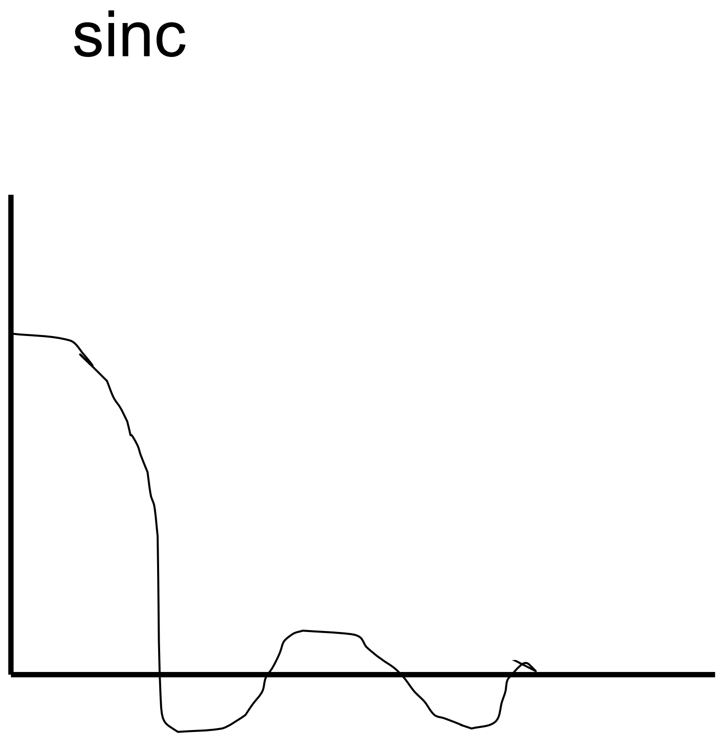
$-b/2 < x < b/2$

otherwise

$$\Im\{\Pi(x)\} = hb \sin(\pi ub)/(\pi ub)$$
$$= hb \sin c(ub)$$

Useful Transforms
Tophat





Useful Transforms
sinc

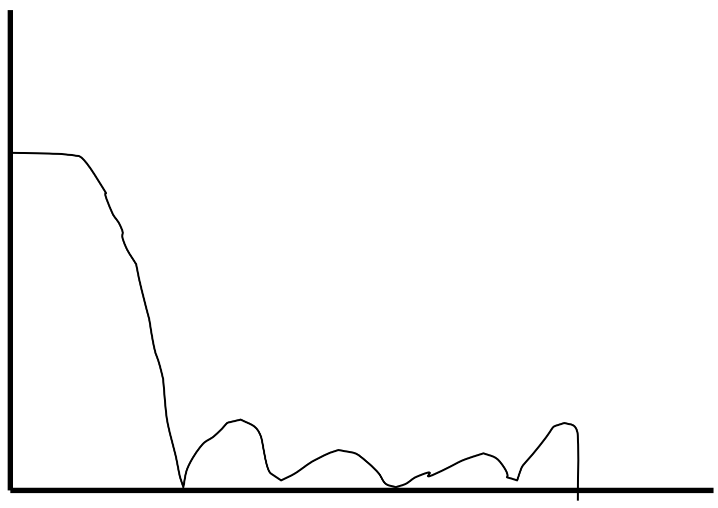
$$\Im(\sin c(x)) = \Pi(u)$$

$$\Im(\sin c^2(x)) = \Lambda(u)$$

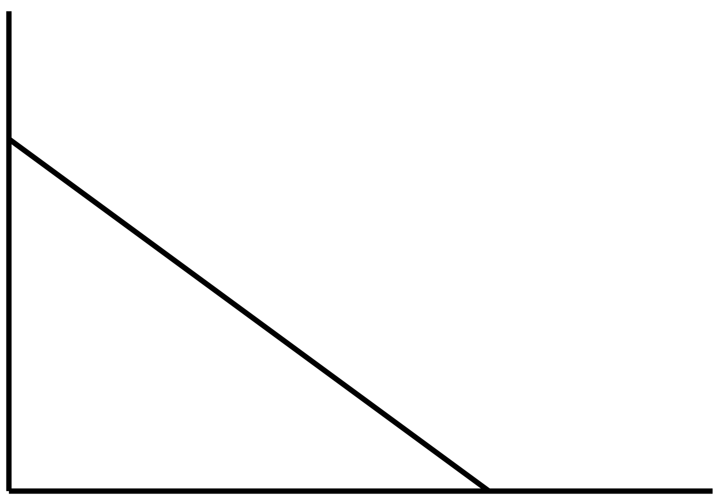
$$\Lambda(x) = \begin{cases} 1 - |x| \ldots\ldots\ldots |x| < 1 \\ 0 \ldots\ldots\ldots\ldots\ldots\ldots |x| > 1 \end{cases}$$

Useful Transforms
Tophat

Sinc^2



Lambda



Useful Transforms
Gaussian

$$f(x) = h \exp(-x^2 / \sigma^2)$$

$$\Im\{f(x)\} = h\sigma\sqrt{\pi} \exp(-\pi^2 u^2 \sigma^2)$$

You can’t deGauss a Gauss!

examples

Rectangular slit width **a**:

$$A(x_0) \propto \mathrm{sinc}(x_0 a / \lambda D) = \frac{\sin(\pi x_0 a / \lambda D)}{\pi x_0 a / \lambda D}$$

examples

D=diameter

Circular aperture diameter :

$$A(\rho_0) \propto \frac{J(\pi\rho_0\Delta/\lambda D)}{\pi\rho_0\Delta/\lambda D}$$

J=Bessel function

$$[I(\rho_0) = |A(\rho_0)|^2 \equiv Airy\ Profile]$$

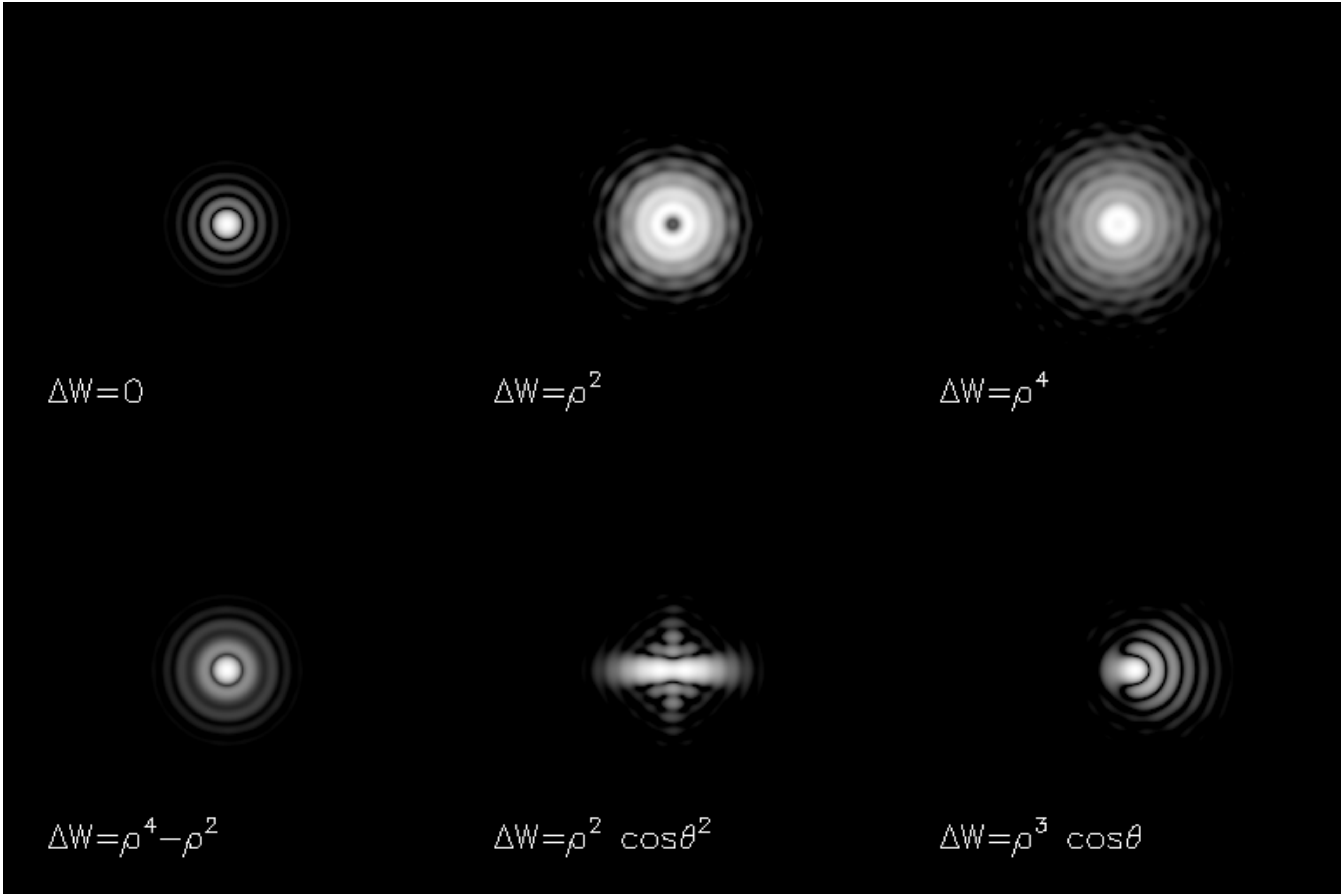
Optical Transfer function

$$OTF(u,v) = \Im\{PSF\}$$
$$= \Im\{\left|\Im\{f(x,y)\}\right|^2\}$$
$$= \int_{-\infty}^{\infty} f(x',y')f^*(x'+x,y'+y)dx'dy'$$

Frequency response function
Tells us how well a lens can pass information as a function of increasing spatial frequency

properties

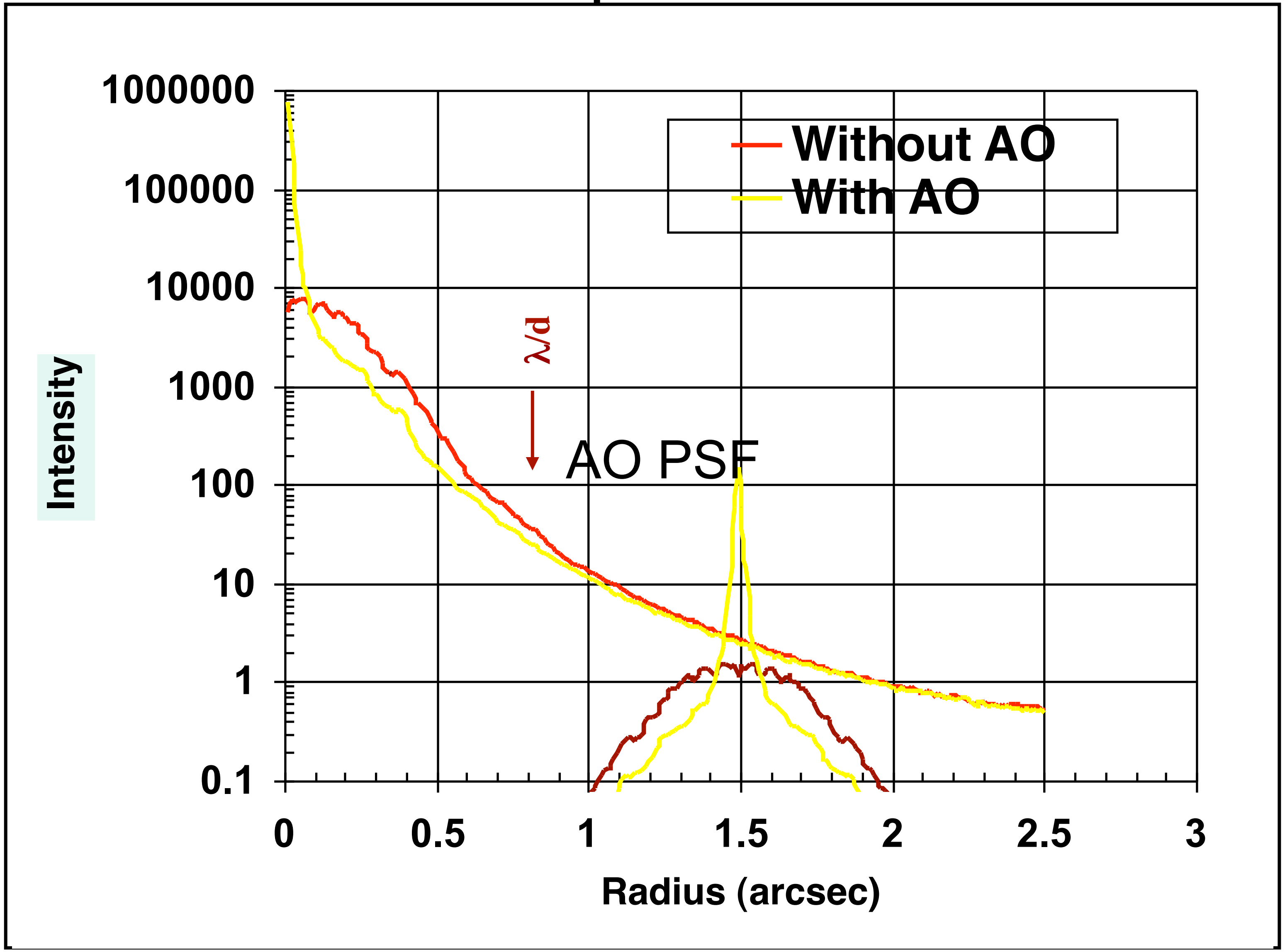
Property	**Function**	**Fourier Transform**
	$f(t)$	$\hat{f}(\omega)$
Inverse	$\hat{f}(t)$	$2\pi\, f(-\omega)$
Convolution	$f_1 \star f_2(t)$	$\hat{f}_1(\omega)\, \hat{f}_2(\omega)$
Multiplication	$f_1(t)\, f_2(t)$	$\frac{1}{2\pi}\, \hat{f}_1 \star \hat{f}_2(\omega)$
Translation	$f(t - t_0)$	$e^{-it_0\omega}\, \hat{f}(\omega)$
Modulation	$e^{i\omega_0 t}\, f(t)$	$\hat{f}(\omega - \omega_0)$
Scaling	$f(\frac{t}{s})$	$\lvert s \rvert\, \hat{f}(s\,\omega)$
Time derivatives	$f^{(p)}(t)$	$(i\omega)^p\, \hat{f}(\omega)$
Frequency derivatives	$(-it)^p\, f(t)$	$\hat{f}^{(p)}(\omega)$
Complex conjugate	$f^*(t)$	$\hat{f}^*(-\omega)$
Hermitian symmetry	$f(t) \in \mathbb{R}$	$\hat{f}(-\omega) = \hat{f}^*(\omega)$

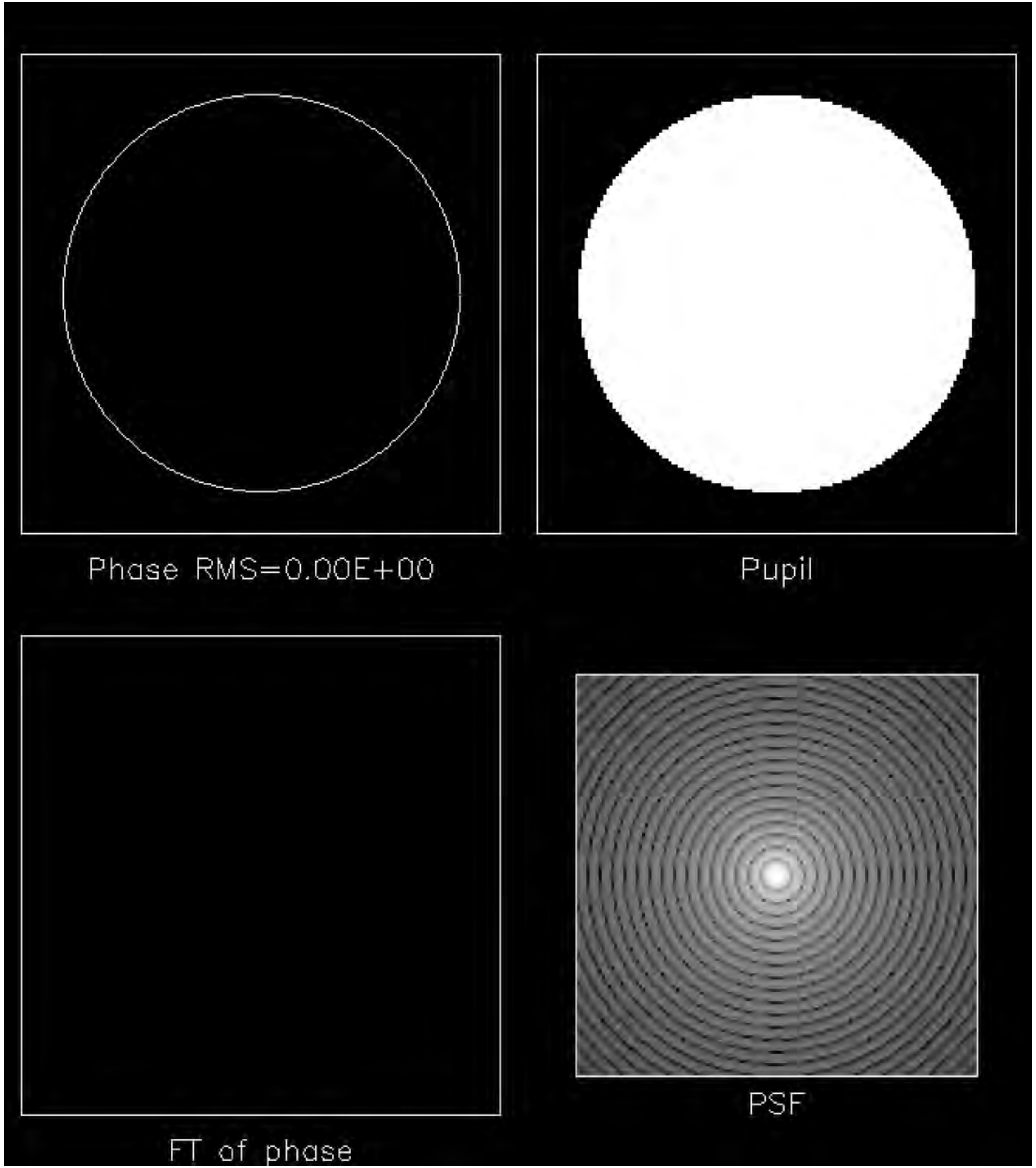

ΔW=0
ΔW=ρ2
ΔW=ρ4
ΔW=ρ4−ρ2
ΔW=ρ2 cosθ2
ΔW=ρ3 cosθ

Example: Coronography

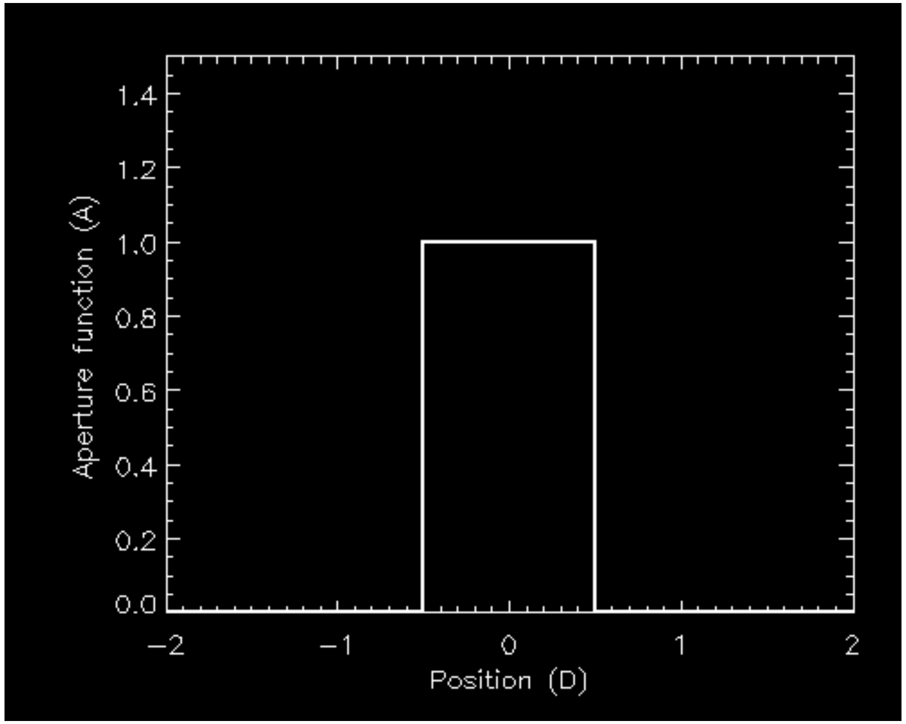
- 10e-6 to 10e-9 surface brightness ratio
- Scattered light generally far brighter than corona
- What to do?

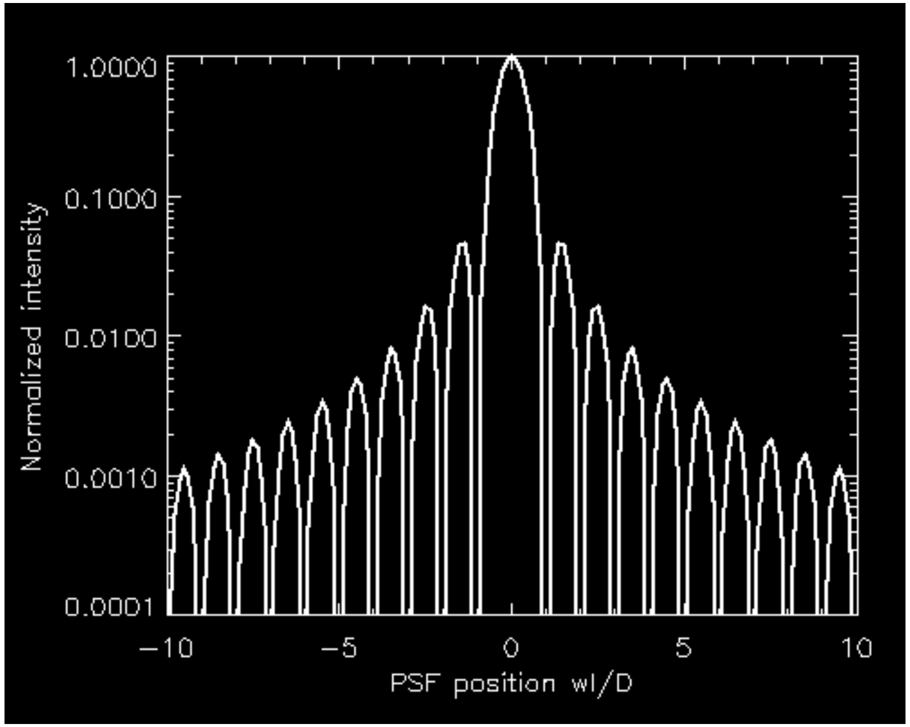
Even worse when we try to image extra-solar planets



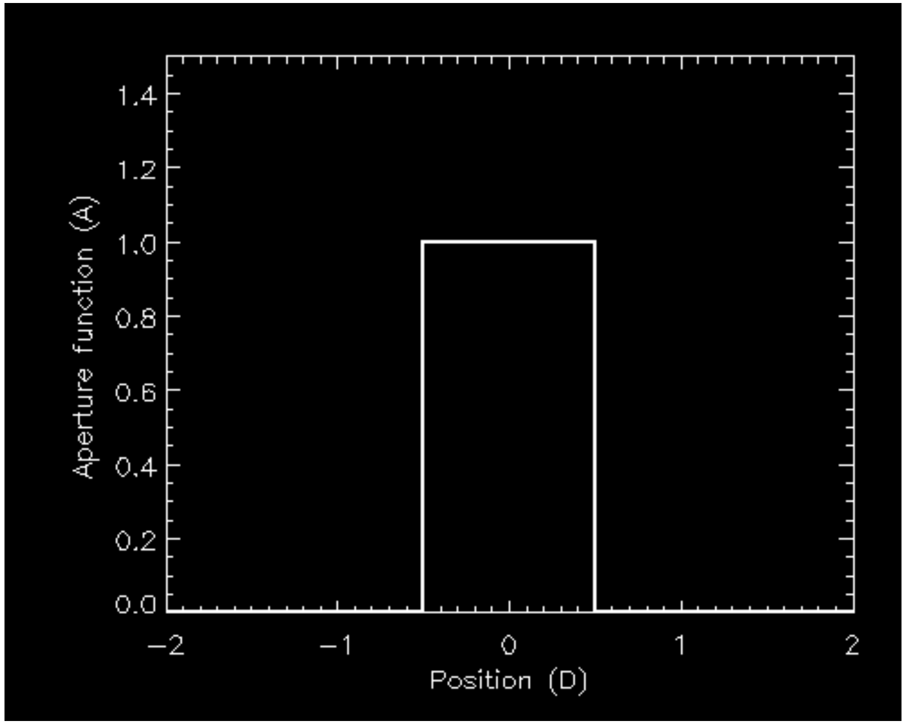

Phase RMS=0.00E+00
Pupil
FT of phase
PSF

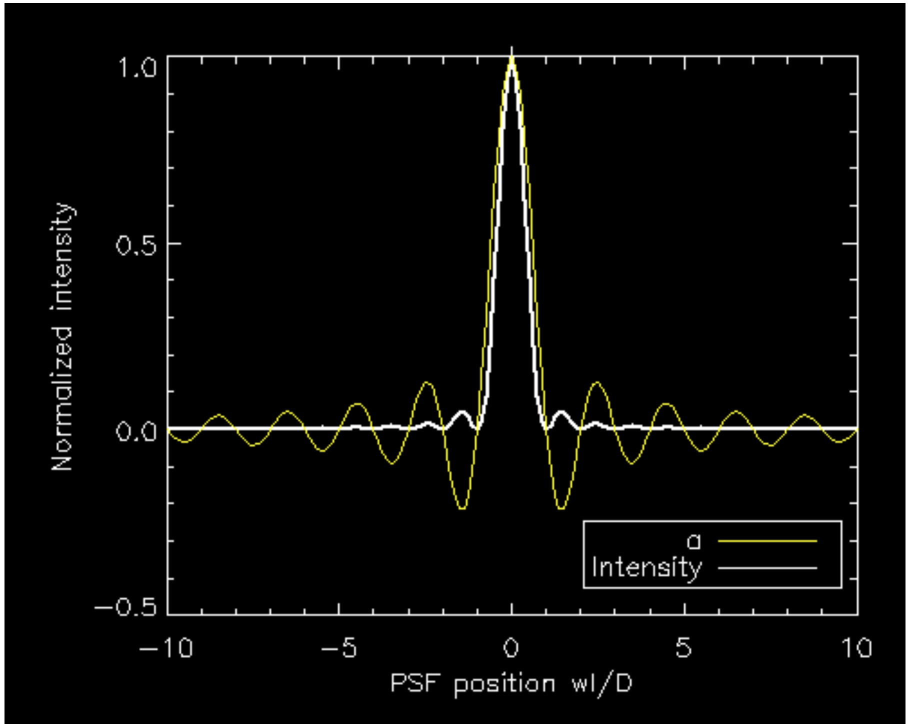
$aa^* = |FT(A)|^2$ is the diffraction term

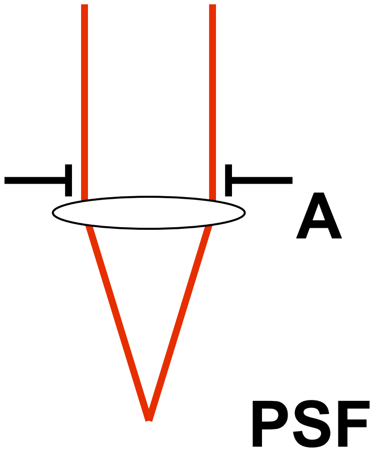




How can we control diffraction?







$$PSF = aa^* = |FT(A)|^2$$

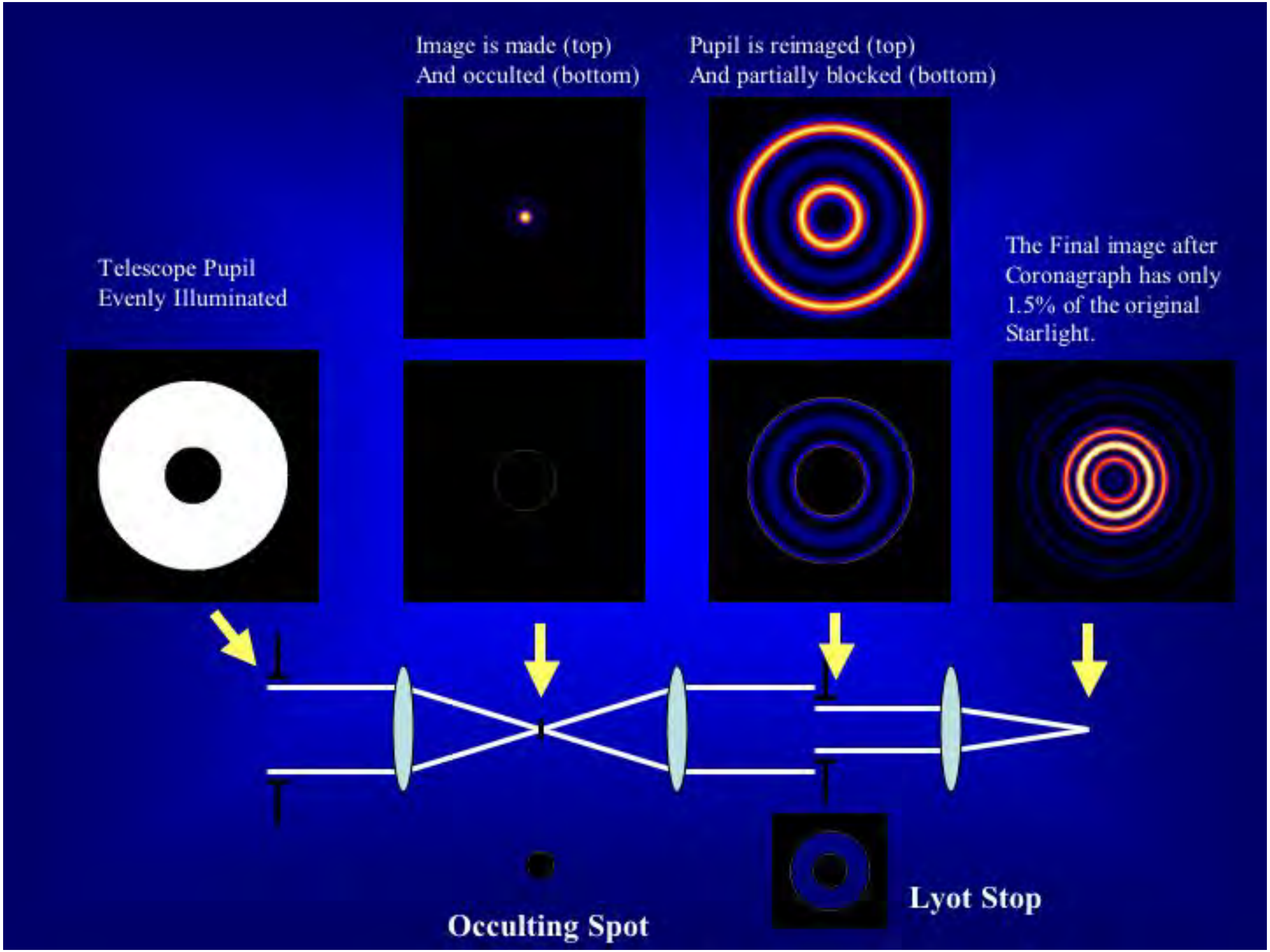
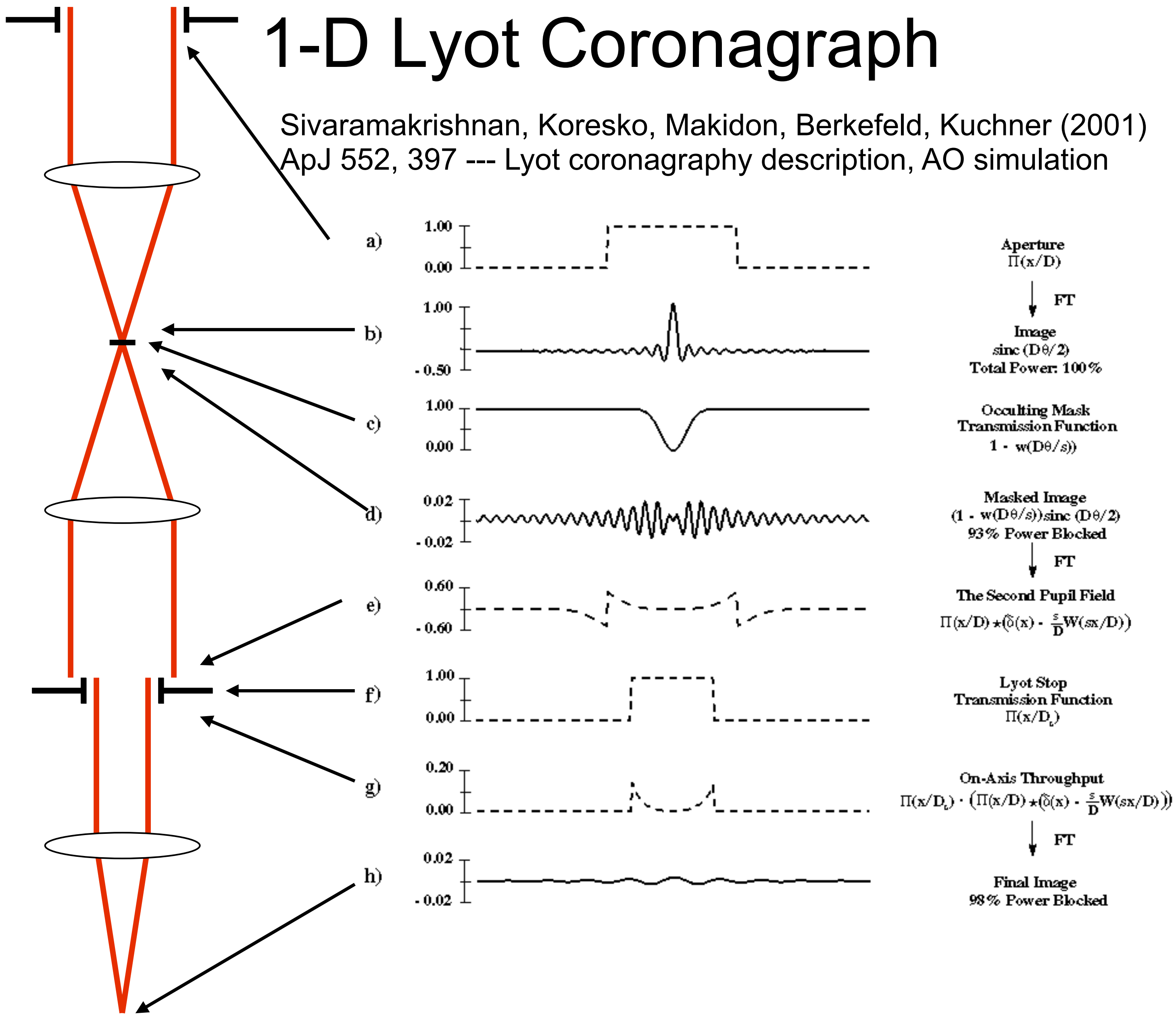
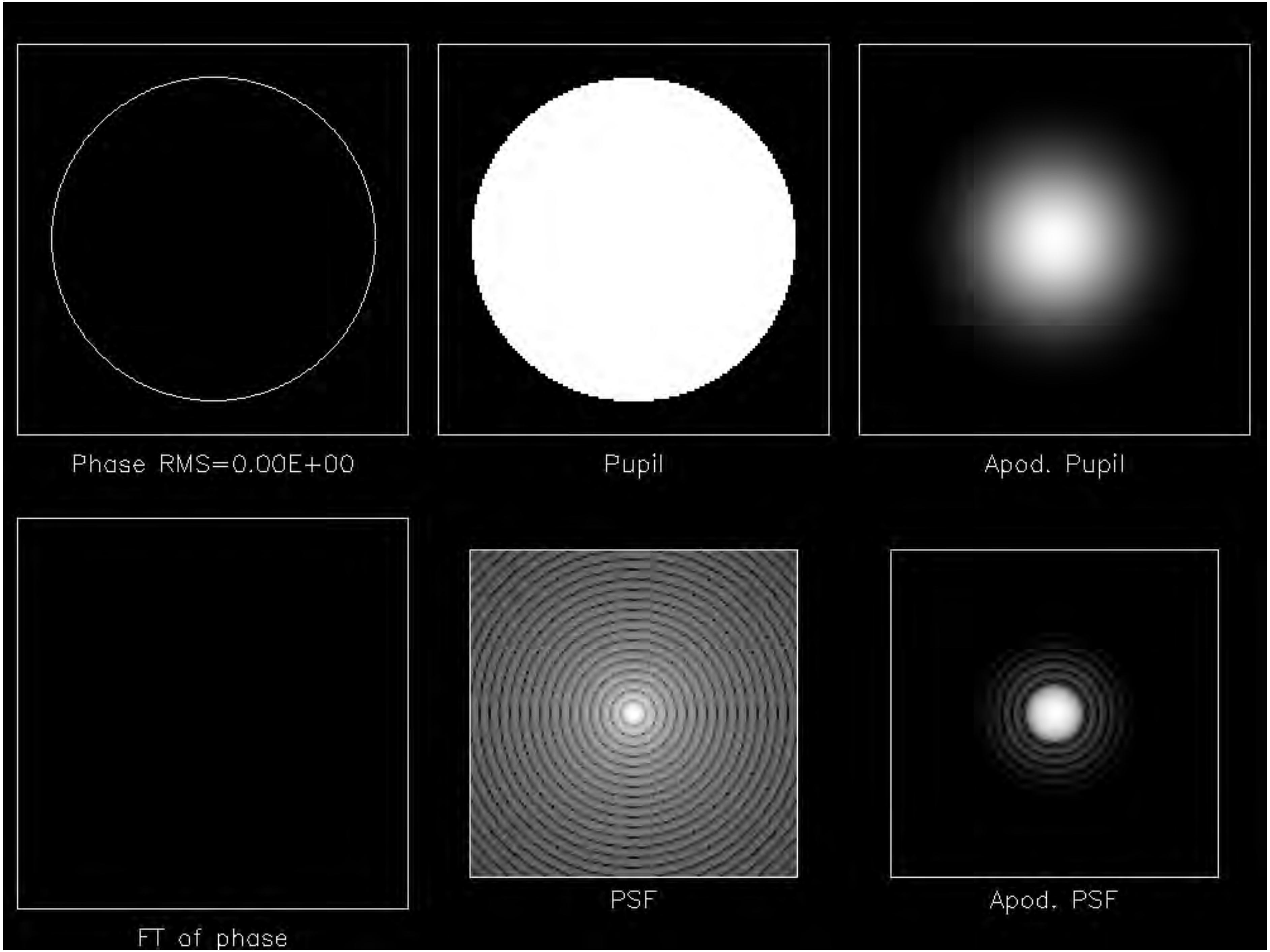


Image is made (top)
And occulted (bottom)
Pupil is reimaged (top)
And partially blocked (bottom)
Telescope Pupil
Evenly Illuminated
The Final image after
Coronagraph has only
1.5% of the original
Starlight.
Occulting Spot
Lyot Stop

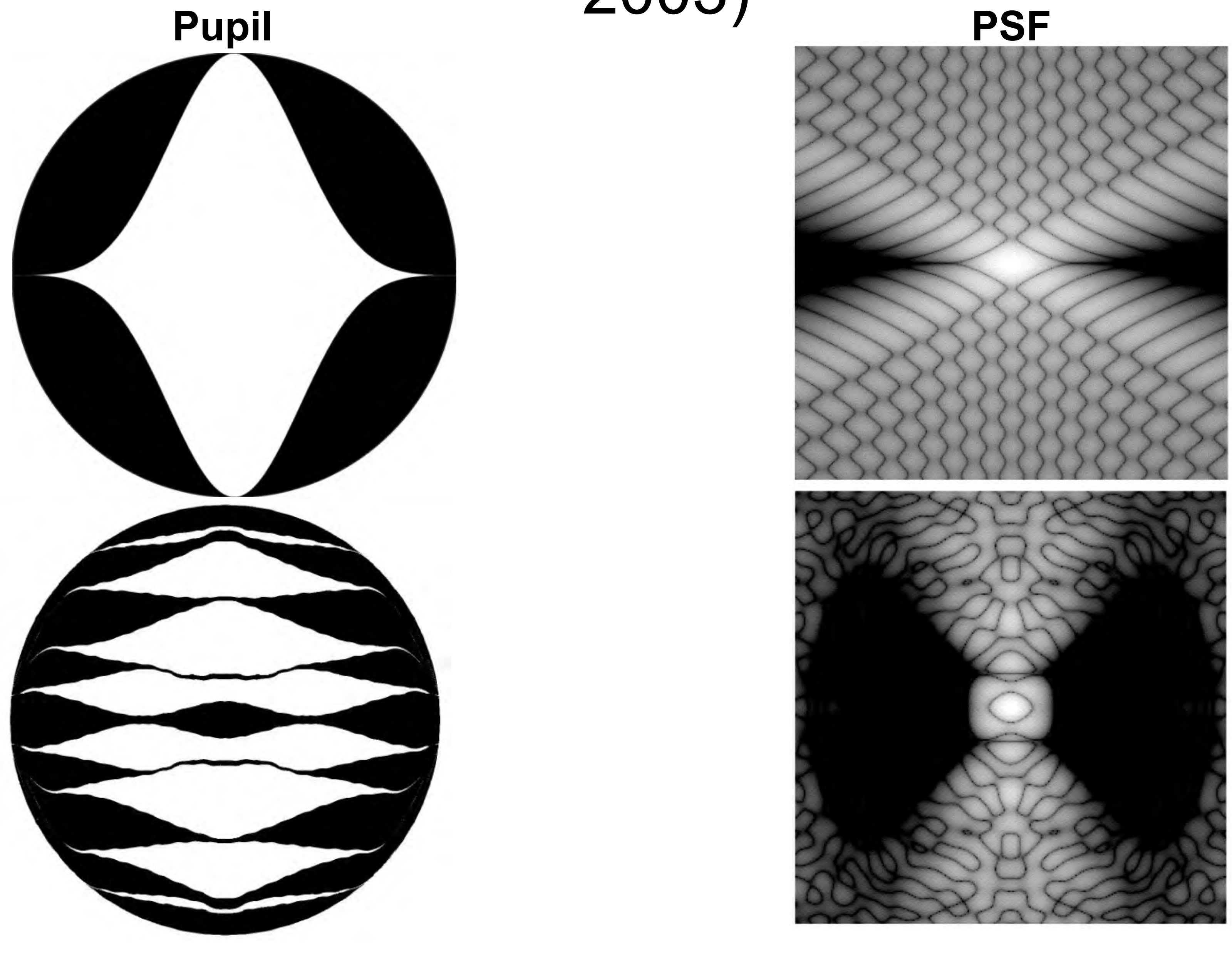
1-D Lyot Coronagraph

Sivaramakrishnan, Koresko, Makidon, Berkefeld, Kuchner (2001)
ApJ 552, 397 --- Lyot coronagraphy description, AO simulation




Phase RMS=0.00E+00
Pupil
Apod. Pupil
FT of phase
PSF
Apod. PSF

Shaped-pupil coronagraphs (Kasdin et al. 2003)

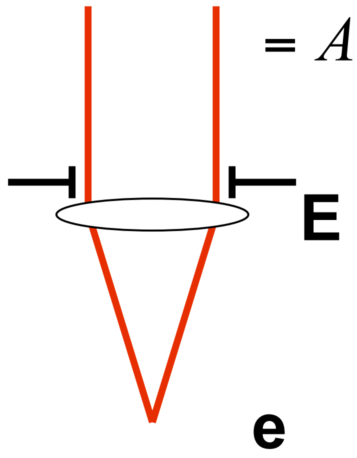


High-contrast AO PSF

- We can use a Taylor expansion of the electric field to determine the PSF in the regime where phase errors are small
- (x,y) = pupil plane coordinates
- (η,ξ) = focal plane coordinates
  - Spatial frequency 1/d <=> angular scale λ/d
- Upper case / lower case = fourier transform pairs
  - Upper case for pupil plane
- E,e = electric field
- P,p = PSF (intensity)
- A = aperture
- Φ = phase
- a,ϕ = fourier transforms of above

$$E(x,y) = A(x,y)e^{i\Phi(x,y)}$$

$$= A(1 + i\Phi - \frac{\Phi^2}{2} + \ldots)$$



Second-order PSF expansion (Sivaramakrisnan et al. 2002)

$$e(\xi,\eta) = FT(A(x,y)e^{i\Phi(x,y)})$$

$$= FT(A + iA\Phi - \frac{A\Phi^2}{2} + \ldots)$$

$$= a + i(a * \phi) - \frac{a * \phi * \phi}{2} + \ldots$$

$$p(\xi,\eta) = \left|e(\xi,\eta)\right|^2$$

$$= (a + i(a * \phi) - \frac{a * \phi * \phi}{2} + \ldots)(a^* - i(a^* * \phi^*) - \frac{a^* * \phi^* * \phi^*}{2} + \ldots)$$

$$= aa^*$$

$$- i[a(a^* * \phi^*) - a^*(a * \phi)]$$

$$+ (a * \phi)(a^* * \phi^*)$$

$$- \tfrac{1}{2}\left[a(a^* * \phi^* * \phi^*) + a^*(a * \phi * \phi)\right]$$

$$+ \ldots$$

PSF terms

$$p = p_0 + p_1 + p_2 + \ldots$$

$$p_0 = aa^*$$

$$p_1 = -i[a(a^* * \phi^*) - a(a * \phi)]$$

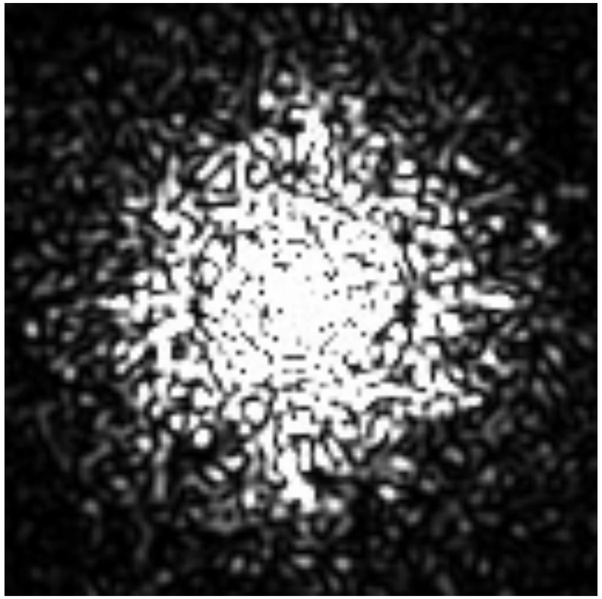
$$= 2\,\mathrm{Im}[a(a^* * \phi^*)]$$

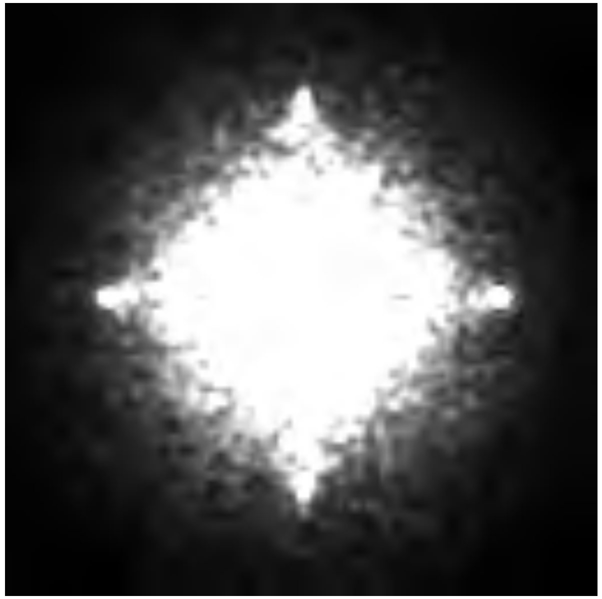
$$p_2 = (a * \phi)(a^* * \phi^*)$$

$$-\tfrac{1}{2}\left[a(a^* * \phi^* * \phi^*) + a^*(a * \phi * \phi)\right]$$

- Diffraction pattern term
- Pinned speckle term
  - Antisymmetric
  - Traces the diffraction pattern; vanishes when diffraction is negligible
- Halo term
  - ~=$|f|^2$
  - Symmetric
- Strehl term
  - Removes power from PSF core

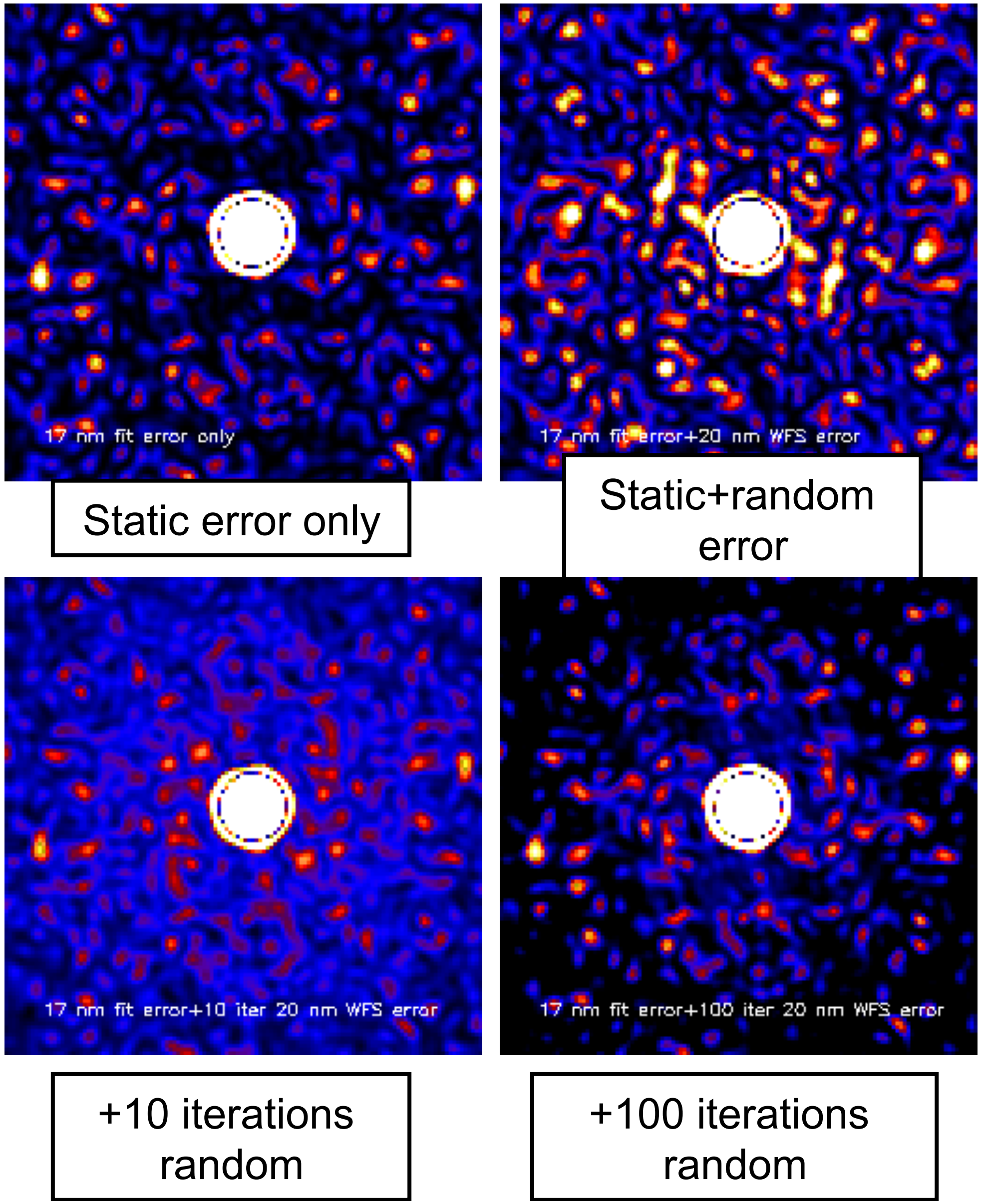
Residual PSF speckles limit high-contrast imaging but for atmospheric sources smooth out with time





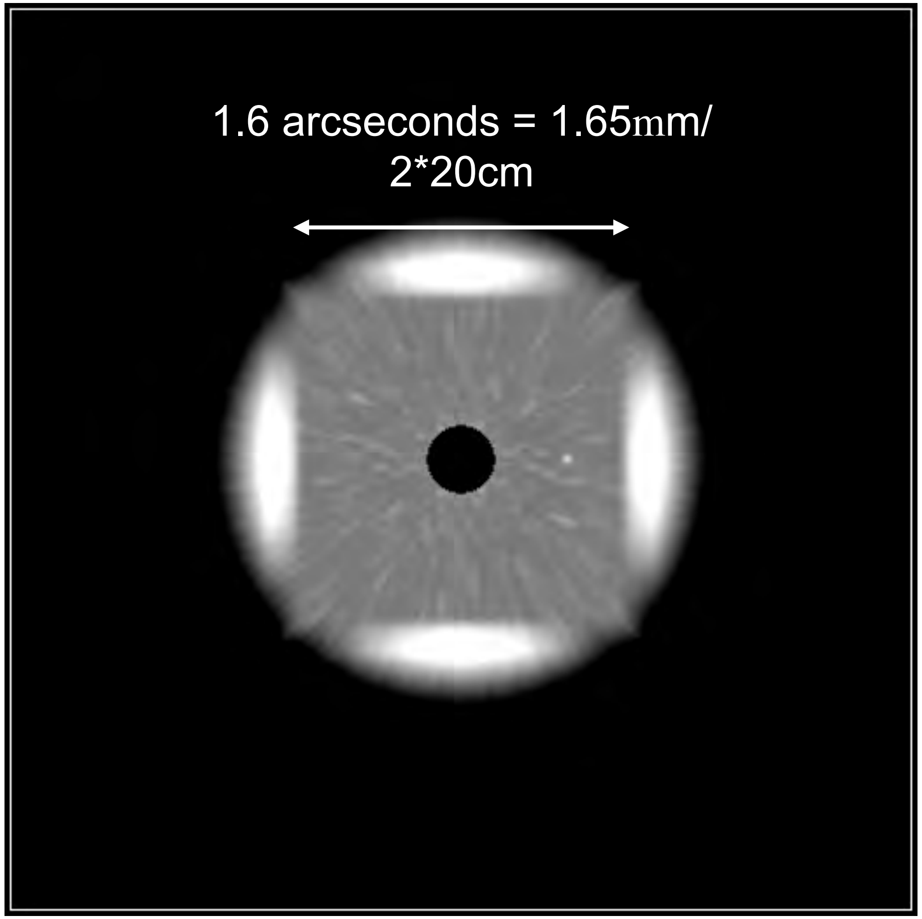
Static errors limit high-contrast sensitivity of current AO

- Multiple speckle sources do not add but instead produce new speckle pattern
- In a long-exposure image with some static errors, PSF returns to original static pattern
- Static error sources dominate sensitivity for all current AO systems



eXtreme Adaptive Optics Planet Imager: XAOPI

- A ~4096 actuator AO system for an 8-10m telescope
- Science goals:
  - direct detection of extrasolar planets through near-IR emission at contrast of 10^7-10^8
  - characterization of circumstellar dust
- Status: 2003-4 Conceptual design study
  - System could be deployed in 2007-8
- System is intended to be facility-class
  - A wide variety of high-contrast science programs are available
  - System will operate on targets brighter than m_R~7-10



Sources and properties of wavefront error

- AO system with actuator spacing *d* can correct spatial frequencies up to 1/2*d* (Nyquist)
- Hence can correct PSF terms out to "control radius" $\lambda/2d$
- Classic atmospheric fitting error (high frequency)
  - Primarily scatters light to large radii
- Uncorrected telescope aberrations (also high frequency)
- Aliasing of high-frequency errors to low frequency
  - A major source of scattered light inside control radius
  - Spatially Filtered WFS (Macintosh et al. 2003, Poyneer and Macintosh submitted) corrects this
- Temporal (bandwidth) errors
  - Scatters light close to star, pushes ExAO to fast systems
- AO system measurement noise
  - Other major source of scatted light inside control radius; pushes ExAO to bright stars
- Internal calibration errors