

What was the next big transition after BBNS ?

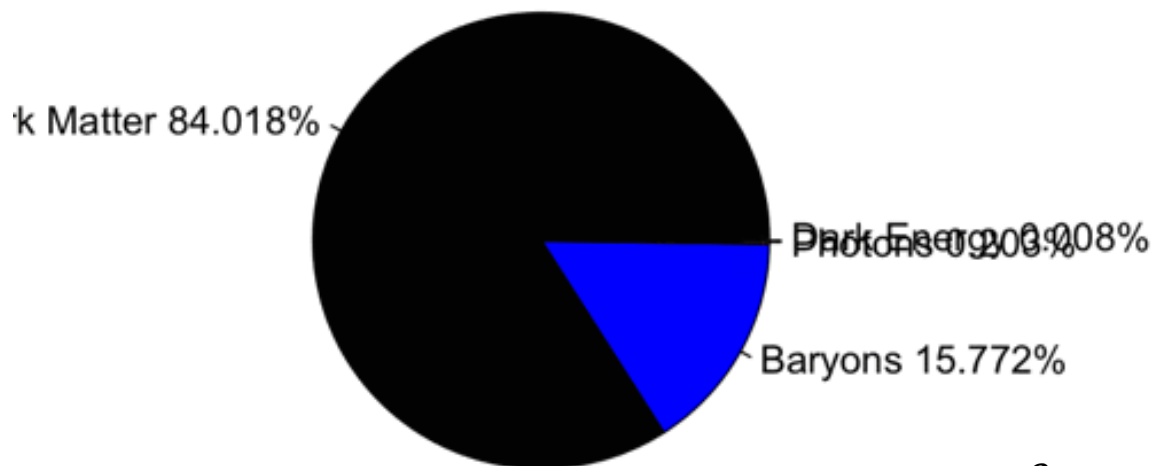
Matter-radiation equality at $z \approx 3600$ (47,000 yrs)

We went over this in the tutorial yesterday, but this gives me a chance to show a video.

Second cross over point (time in Gyr):

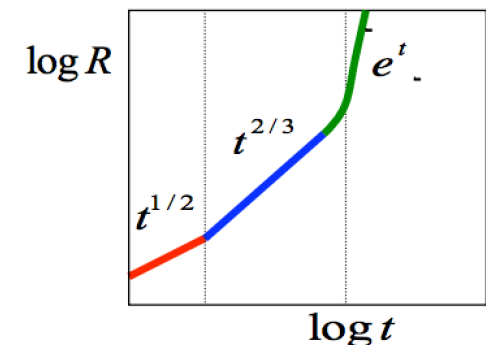
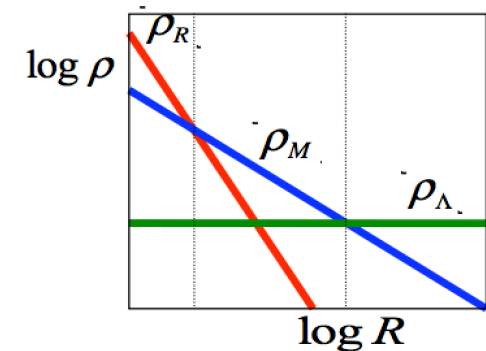
We learn here that matter dominated over radiation and the vacuum from $z=3600$ to $z=0.65$

Time since Big Bang 0.1



First cross over point:

$$\begin{aligned} \text{radiation : } & \rho_R \propto R^{-4} \\ \text{matter : } & \rho_M \propto R^{-3} \\ \text{vacuum : } & \rho_\Lambda = \text{const} \end{aligned}$$

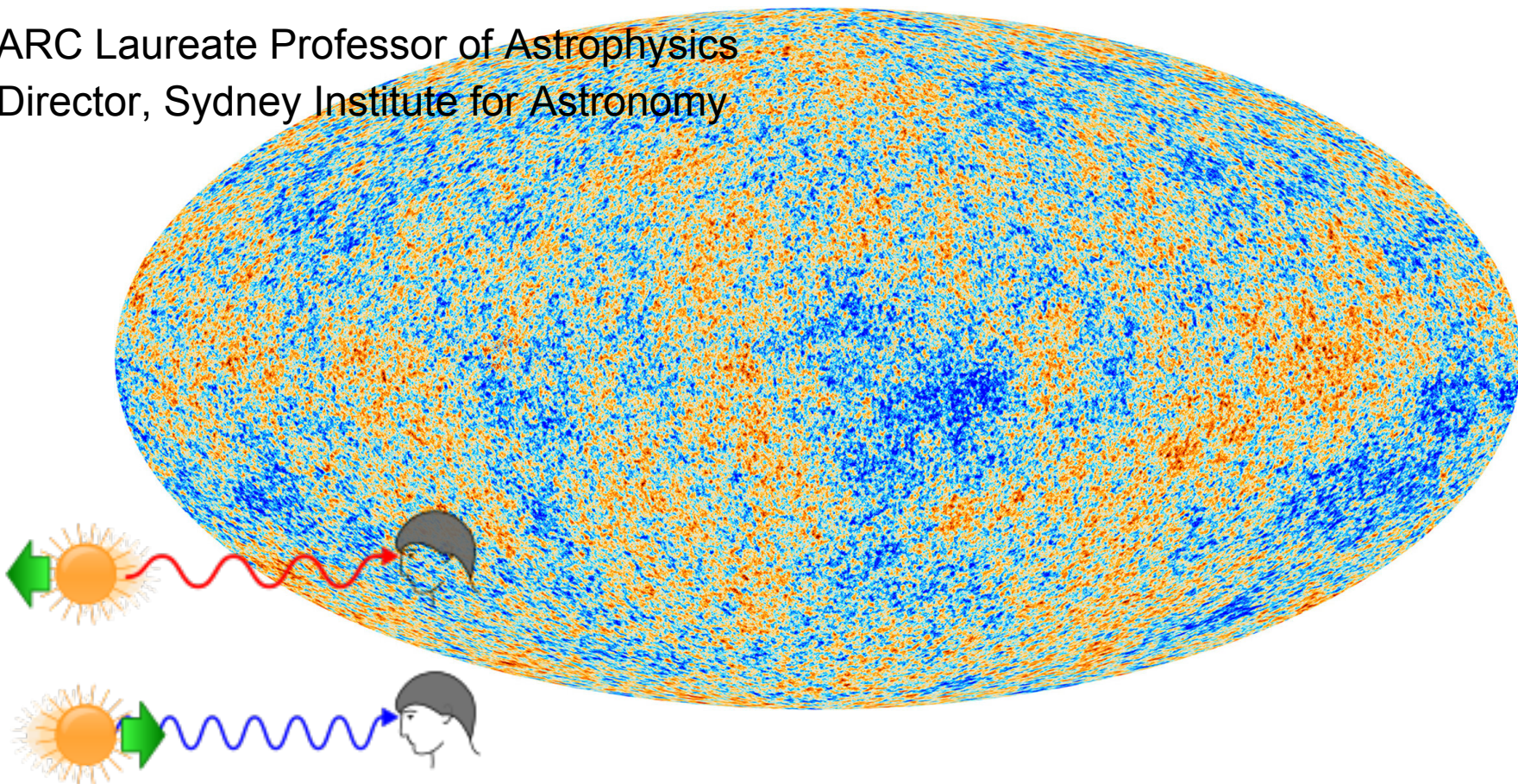


$$\frac{\epsilon_{\text{matter}}}{\epsilon_{\text{radiation}}} = \frac{\Omega_{\text{matter}}}{\Omega_{\text{radiation}}} = \frac{0.3}{0.000084} \approx 3600 = 1/a = 1+z$$

Cosmic microwave background (CMB): fossil of the hot Big Bang

Joss Bland-Hawthorn

ARC Laureate Professor of Astrophysics
Director, Sydney Institute for Astronomy



Epochs leading up to the CMB

event	redshift	temperature (K)	time (megayears)
radiation-matter equality	3570	9730	0.047
recombination	1370	3740	0.24
photon decoupling	1100	3000	0.35
last scattering	1100	3000	0.35

Recombination epoch:

Universal gas goes from **ionized** to **neutral** H, He like a warm fog

After this epoch, universe filled with H, He fog. Photons can penetrate this at longer wavelengths, e.g. CMB photons, but not UV photons because these ionize H, He. This fog lifts **much later** due to later sources. We call

electrons in atoms, photons move freely now since easily scattered by electrons

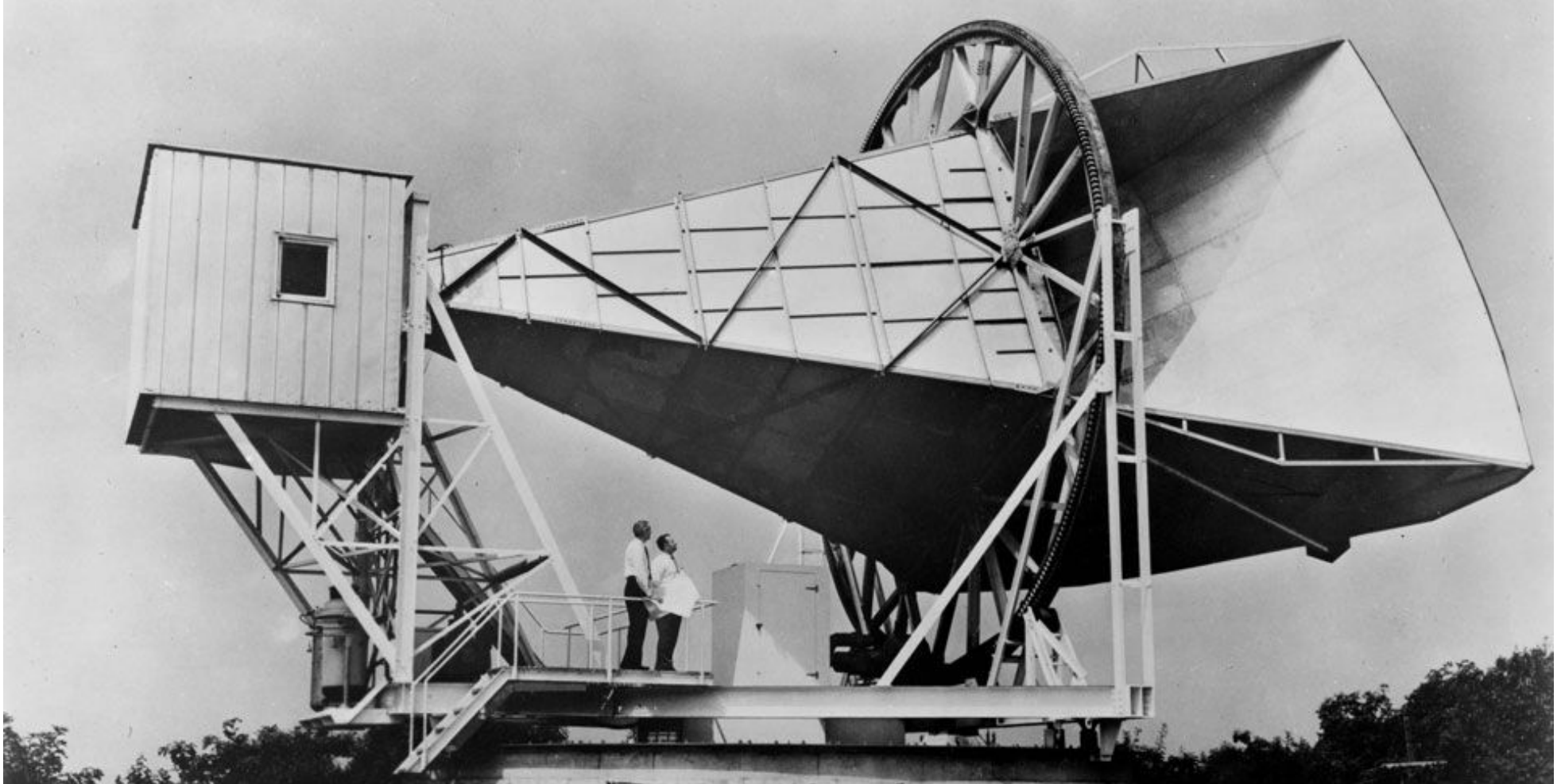
...try shining a torch on a foggy night. You make the beam even brighter to see further, but it does not work.

Photons just get scattered so strongly, the back reflection gets brighter, and your vision is even worse.





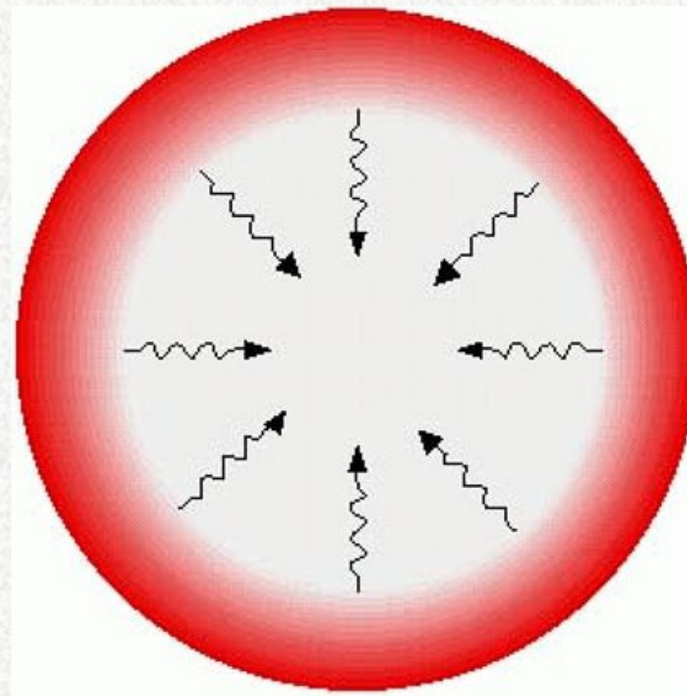
Discovery of cosmic microwave background (CMB)



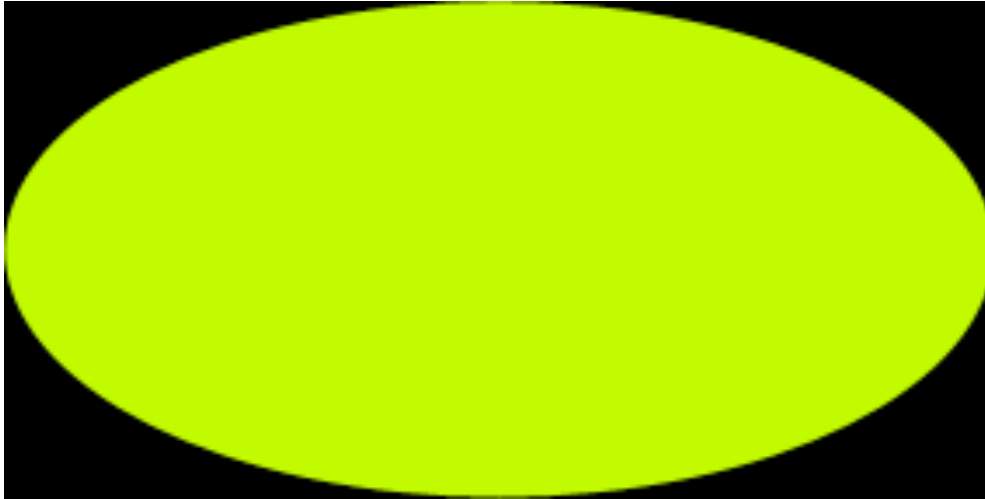
Penzias & Wilson 1965: but it was Australian John Bolton (USyd/CSIRO) who told them what to do and how to do it. He was a brilliant engineer who built Parkes, etc.

We see photons today from last scattering surface
when the universe was just 400,000 years old

The temperature of
the cosmic microwave
background (CMB) is
very nearly the same
in all directions.



Temperature T , BB photon energy $\propto kT$



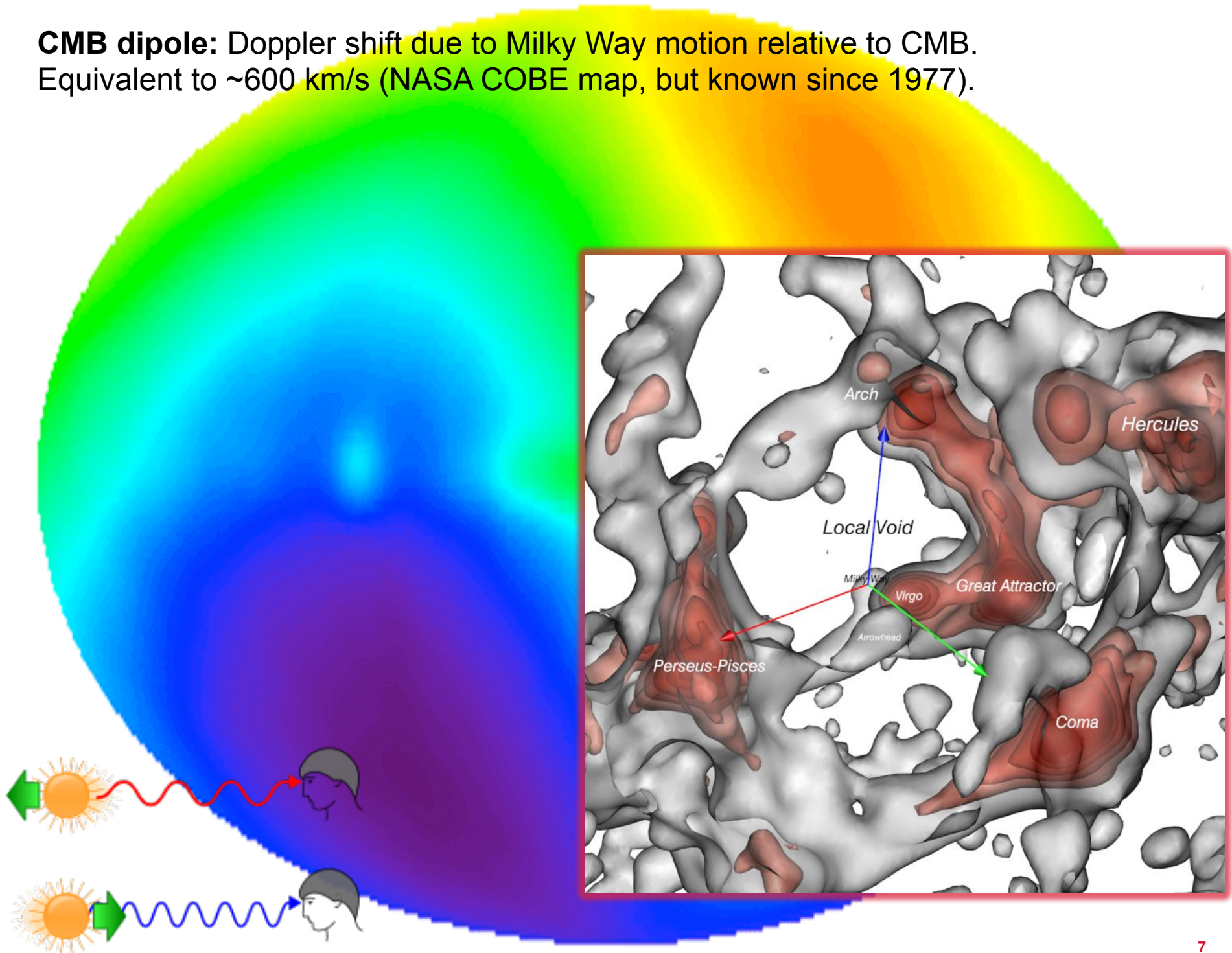
If we average noisy measurements in all directions to confirm uniform, constant temperature since Universe isotropic, is that what we see?

In fact, not until other effects are taken into account.

$$\langle T \rangle = \frac{1}{4\pi} \int T(\theta, \phi) \sin \theta d\theta d\phi = 2.725 \text{ K}$$

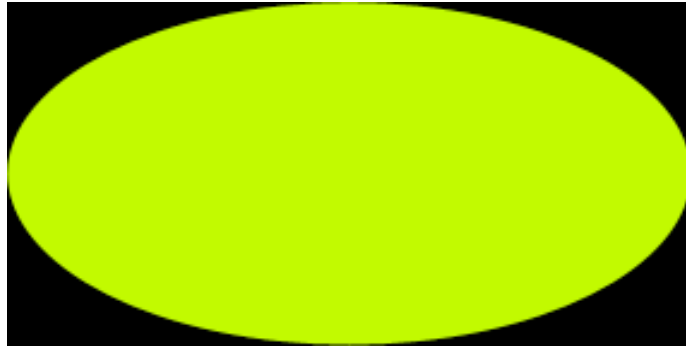
$$\frac{\delta T}{T}(\theta, \phi) \equiv \frac{T(\theta, \phi) - \langle T \rangle}{\langle T \rangle}$$

CMB dipole: Doppler shift due to Milky Way motion relative to CMB.
Equivalent to ~ 600 km/s (NASA COBE map, but known since 1977).



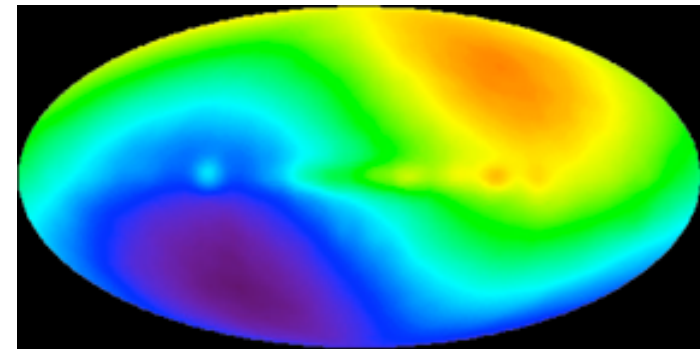
The next amazing epoch was the CMB

Baryons and photons decouple at $z \approx 1089$ (370,000 yrs)



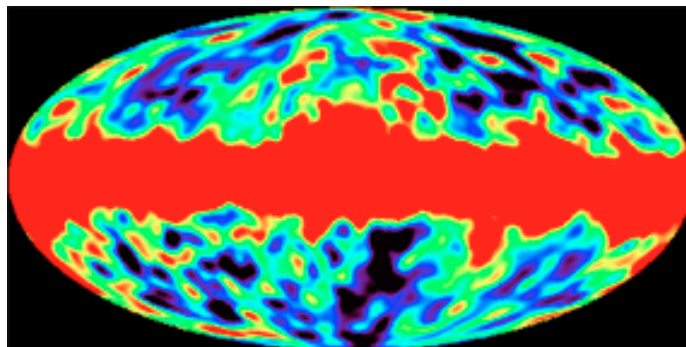
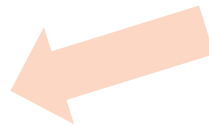
1965

Simple view of the CMB (low contrast).



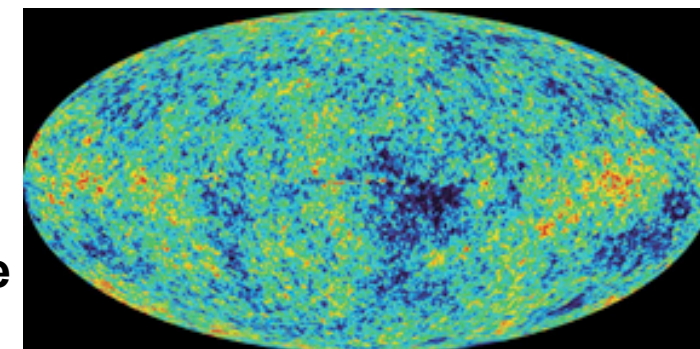
1977

The CMB dipole (our motion in the Universe)



1989

Milky Way structure in the microwave sky



2010

The CMB temperature fluctuations

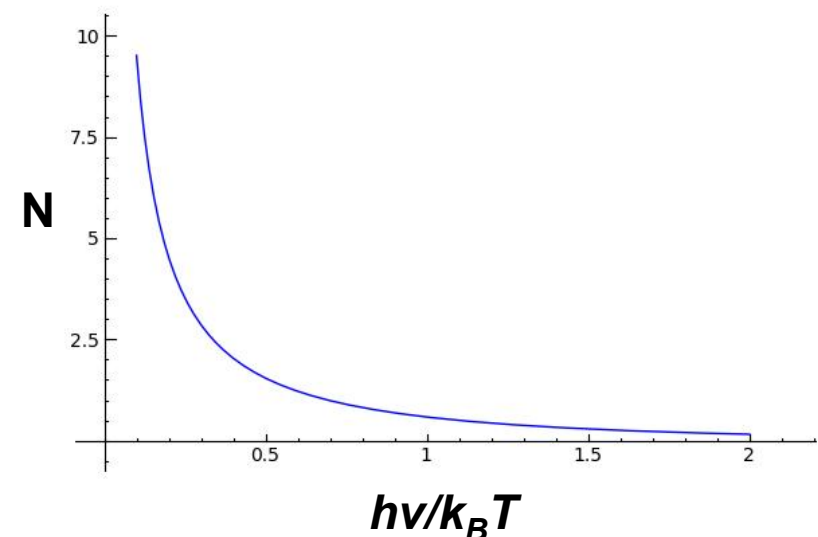
$$\left\langle \left(\frac{\delta T}{T} \right)^2 \right\rangle^{1/2} = 1.1 \times 10^{-5}$$

Blackbody distributions

- › If particles are frequently interacting with each other, they are **in thermal equilibrium**: forwards - backwards interactions proceed at the same rate.
Opaque: only a tiny fraction of the available photons escape per second.
- › In this case, the average particle numbers and energies do not change. The number of particles of a given energy only depends on temperature, T .
- › For photons, the number vs. energy($=h\nu$) is given by a **Planck** or **Black-body** distribution:

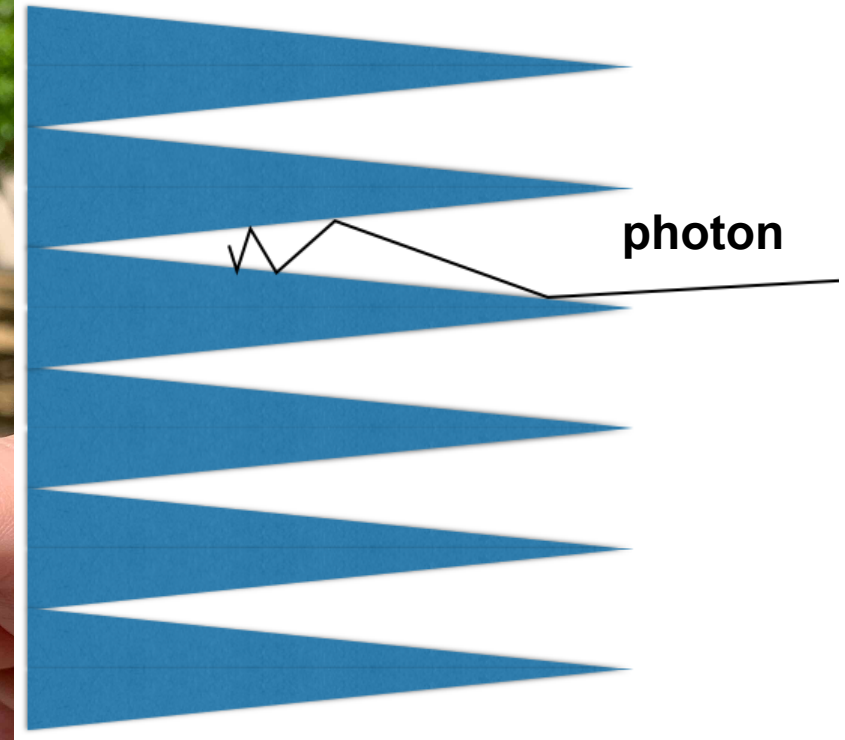
$$N = \frac{1}{\exp(h\nu / k_B T) - 1}$$

- › k_B converts temperature to an energy
- › Much easier to make photons at low energies; the Universe has vastly more.





The perfect blackbody – a stack of razor blades



Seen face on, most photons reflected, say 99%.

Seen edge on, nothing is reflected, **blackbody**.

Blackbody distributions

- From the number of photons of a given energy, it is possible to derive the energy distribution (in this case, volume energy density):

$$\varepsilon(\nu)d\nu = \frac{8\pi h}{c^3} \frac{\nu^3 d\nu}{\exp(h\nu / k_B T) - 1}$$

Peak of distribution is at

$$E_{peak} = h\nu_{peak} \approx 2.8k_B T.$$

Mean of distribution is

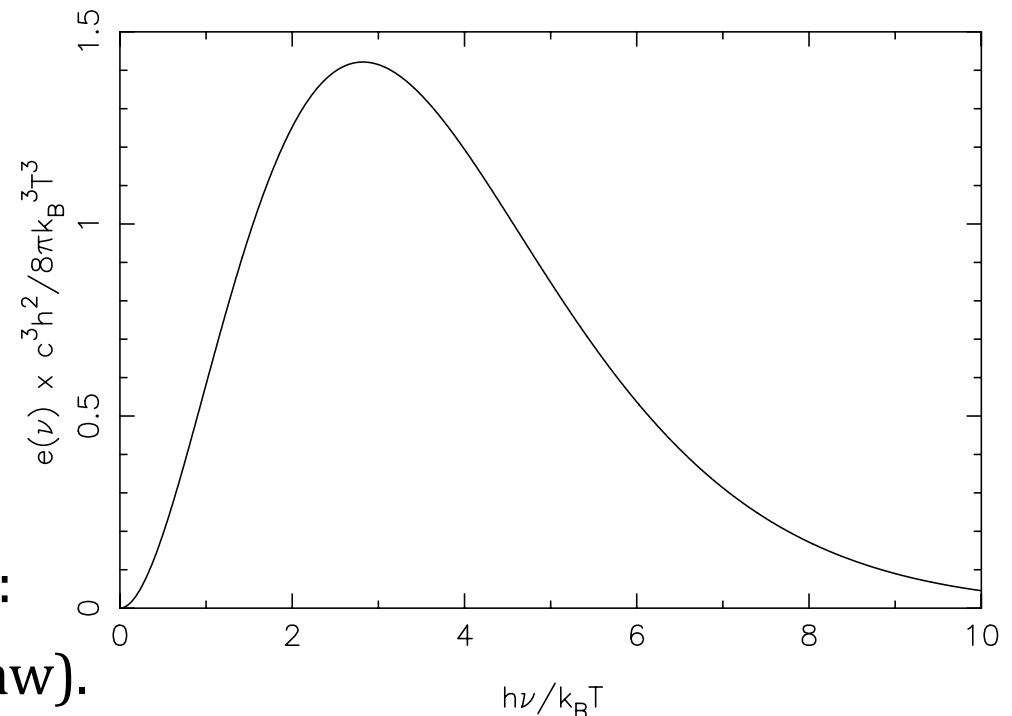
$$E_{mean} = h\nu_{mean} \approx 3k_B T.$$

Integrating the distribution gives:

$$\varepsilon_{rad} = \sigma T^4 \quad (\text{Stefan-Boltzmann law}).$$

Where σ is called the radiation constant:

$$\sigma = \frac{8\pi^5 k_B^4}{15h^3 c^3} = 7.565 \times 10^{-16} \text{ Jm}^{-3} \text{ K}^{-4}$$



$$\int_0^{\infty} \frac{y^3 dy}{e^y - 1} = \pi^4 / 15$$

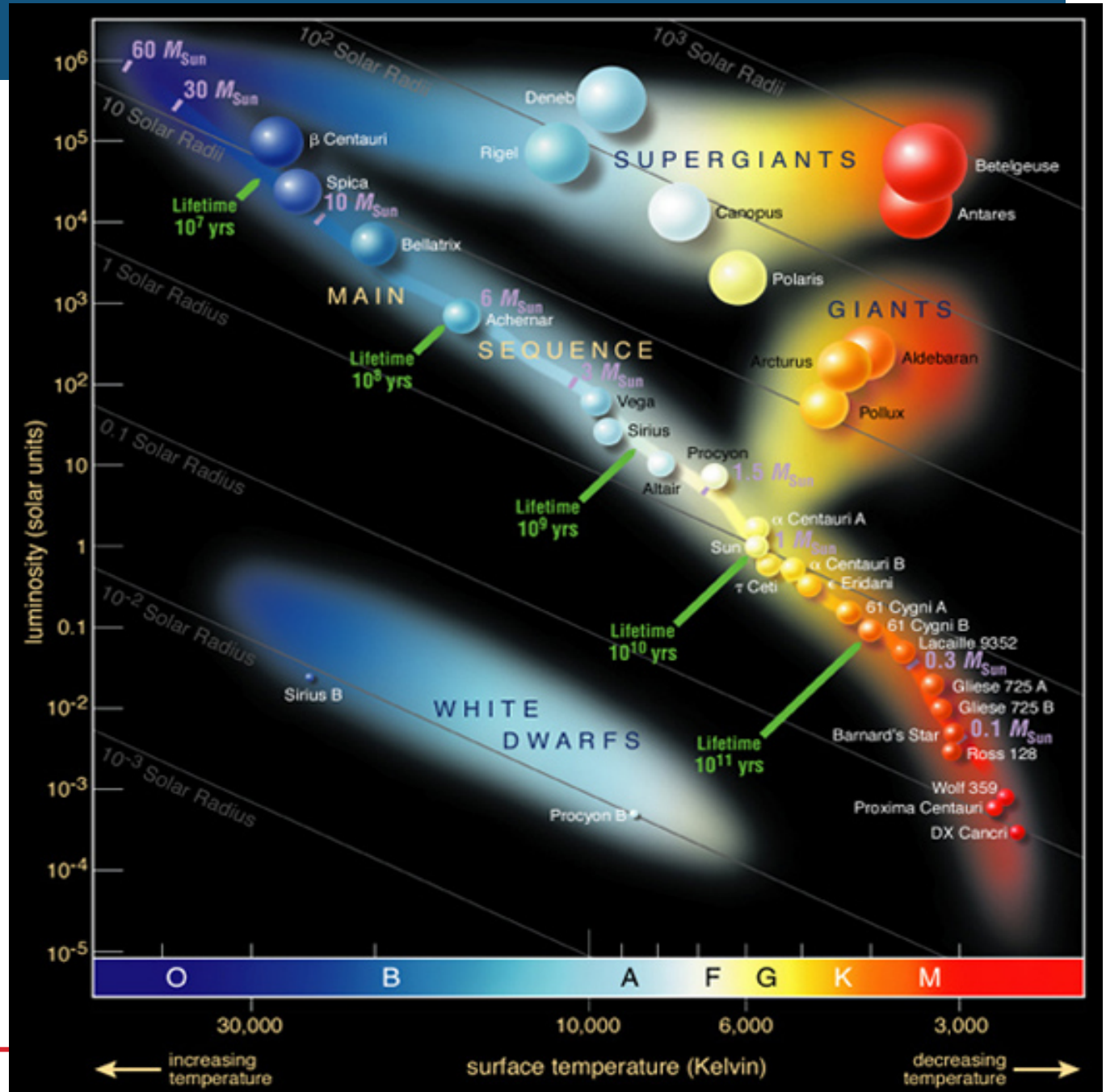
Most stars are approximate to blackbody

Whether you are a big or a small star, if you are cooler, you are redder, and if you are hotter, you are bluer.

You can see this effect across the sky at night.

This is the most famous and arguably the most important diagram in all astrophysics.

(You might wonder why the intrinsic brightness i.e. **luminosity** changes with temperature? Is it just T changing ?)





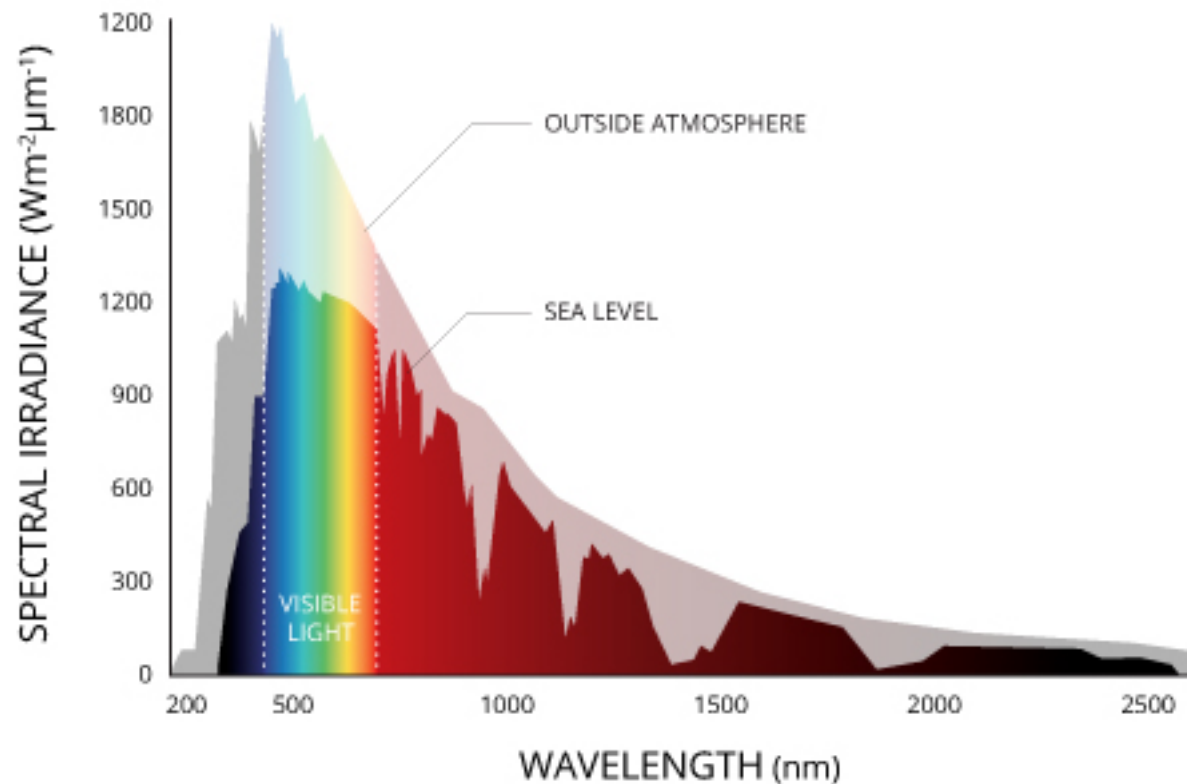
These formulae really work

$$h\nu_{\text{mean}} = 2.8 k_B T = hc / \lambda_{\text{mean}}$$

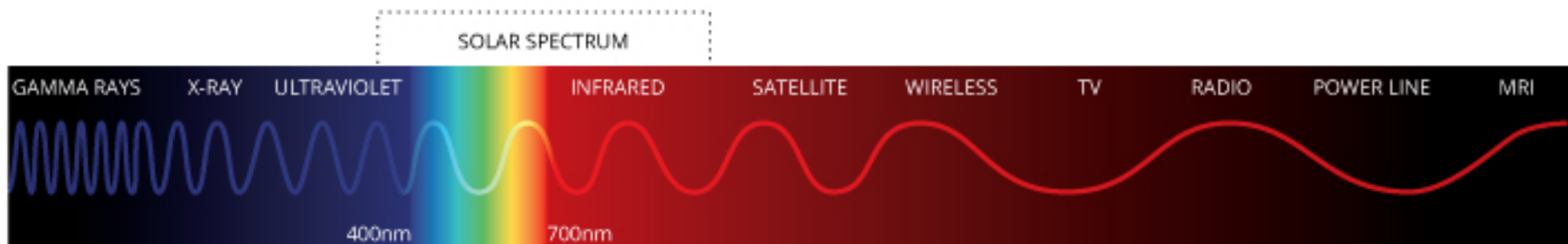
$$\lambda_{\text{mean}} = 10^9 hc / 3k_B T$$

where the big factor converts
from metres to nanometres,
traditional wavelength units.

Now use the nice online
calculator (next slide) to
determine mean wavelength.

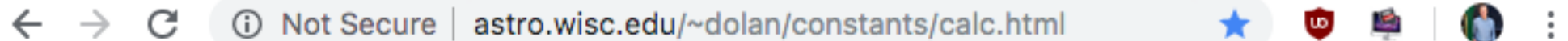


ELECTROMAGNETIC SPECTRUM



Online calculator

This is my favourite **online calculator**. It allows SI or cgs (90% of astronomers use cgs, strangely), and provides lots of natural science and astrophysics units.



This calculator only works if your browser can process JavaScript

Solar Temperature in Kelvin

Units: ☐ CGS ☒ SI

829.739711841635

log10^xln^{e^x}pi^eAC

sinasin^{cos}acos^{tan}atan^C

EE^{sqr}^{sqrt}^{y^x}^{1/x}^{x!}Bksp

789/^{STO}^{STO2}

456^x^{MR}^{MR2}

123⁻^{M+}^{M2+}

0^{+/-}[.]⁺⁼⁽⁾

Physical Constants

c^k^G^{m_e}^{m_p}^{amu}^{eV}^{alpha}

^h^e^R^{m_n}^{m_H}^{N_A}^a^{sigma}

Astronomical Constants

yr^{pc}^{M_{sun}}^{R_{sun}}

^{AU}^{ly}^{L_{sun}}^{T_{sun}}

Planetary Data

Mercury[↓]^m^r^p^a^e

The cosmic microwave background (CMB)

- › We know that

$$\rho_{rad} c^2 = \varepsilon_{rad} = \sigma T^4$$

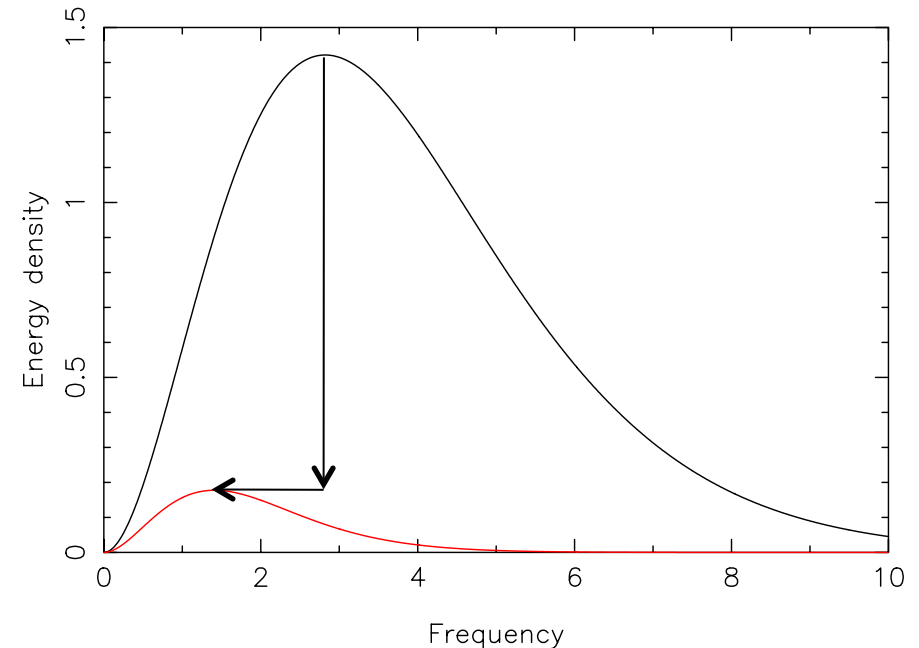
but recall that ρ_{rad} evolves as

$$\rho_{rad} \propto \frac{1}{a^4}$$

which implies that:

$$T \propto \frac{1}{a}$$

- › BB spectrum maintains its shape as T changes.
- › Can maintain a thermal distribution, as long as some time in the past the system was in thermal equilibrium.



$$\varepsilon_\nu d\nu = \frac{8\pi h}{c^3} \frac{\nu^3}{\exp(h\nu/k_B T) - 1} d\nu$$

with expansion: $T \downarrow \propto \nu \downarrow$

Shape dependent on exponential term only

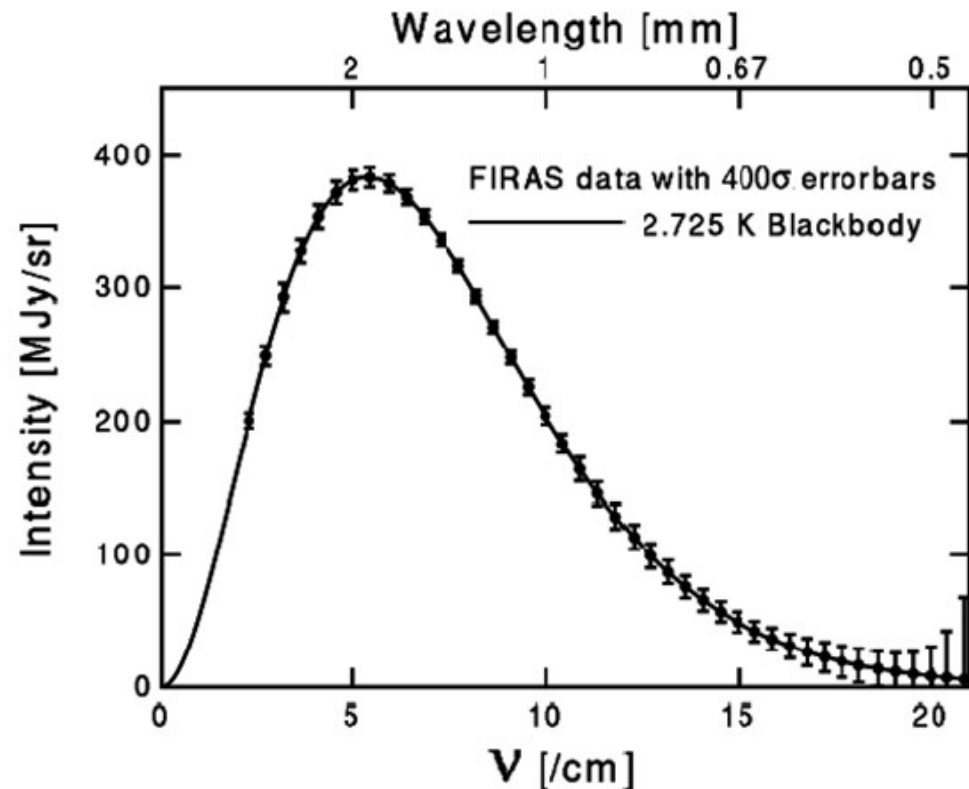
$$\text{i.e., } \varepsilon_{\nu_1} d\nu_1 = a^4 \varepsilon_{\nu_0} d\nu_0$$

if a is the expansion factor between ν_1 and ν_0

cosmic microwave background measurements

- › Cosmic Background Explorer (COBE) satellite, launched in 1989, finds $T_0 = 2.725$ K.
- › **Perfect blackbody spectrum.**

On 13 Jan 1990, this “first light” plot shown at AAS by Dr. John Mather (later Nobel) – he got a standing ovation! The first pure blackbody spectrum, as expected from Big Bang. Wow!



- › Integrating the **blackbody function**, we can find the total energy density, which is

$$\epsilon_{rad} = \sigma T^4 \quad \text{and} \quad \sigma = \frac{8\pi^5 k_B^4}{15h^3 c^3} = 7.565 \times 10^{-16} \text{ Jm}^{-3} \text{ K}^{-4}$$

Measured CMB temperature today is: $T_0 = 2.725 \pm 0.001 \text{ K}$.

$$\text{So } \epsilon_{rad}(t_0) = 4.17 \times 10^{-14} \text{ Jm}^{-3}$$

Mean energy of photon $\approx 3k_B T = 1.13 \times 10^{-22} \text{ J} = 0.00023 \text{ eV}$

(remembering that $k_B = 1.381 \times 10^{-23} \text{ J K}^{-1} = 8.619 \times 10^{-5} \text{ eV K}^{-1}$

and $1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}$)

$$\text{Photon number density} = \frac{\text{energy density}}{\text{mean energy per photon}} \approx 3.69 \times 10^8 \text{ m}^{-3}$$

Can you feel the CMB photons?

Almost a billion CMB photons pass through your body per second, but that cosmic intensity is far too low to cook you from the inside out. A typical microwave oven has 10^{26} photons/sec (1000W) passing through it.

However, go to a TV station where you've not tuned into a station, and you will see this.

Much of the white noise you see here (of course, you probably have the latest LCD display, etc.) is from the CMB.



The photon-to-baryon ratio

Measured baryon density parameter $\Omega_b \approx 0.04$

Critical density: $\rho_{crit}(t_0) = \frac{3H_0^2}{8\pi G} = 0.92 \times 10^{-26} \text{ kg m}^{-3}$

Mass of a typical baryon (e.g. a proton) $\approx 1.67 \times 10^{-27} \text{ kg}$

Baryon number density $\approx 0.92 \times 10^{-26} \times 0.04 / (1.67 \times 10^{-27}) = 0.22 \text{ m}^{-3}$

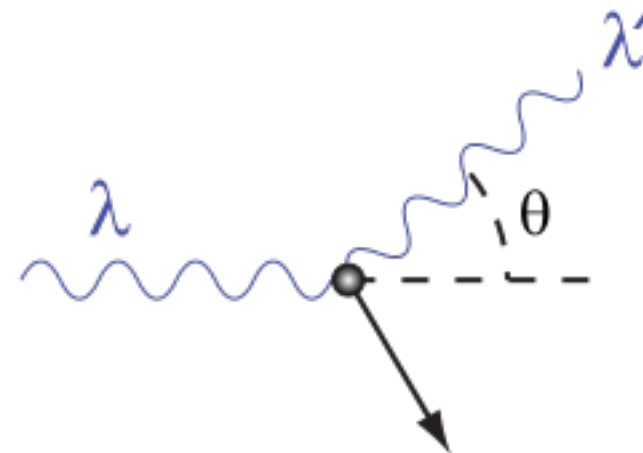
Photon – to – baryon ratio $= 3.69 \times 10^8 / 0.22 = 1.7 \times 10^9$

This is an amazing result in cosmology !

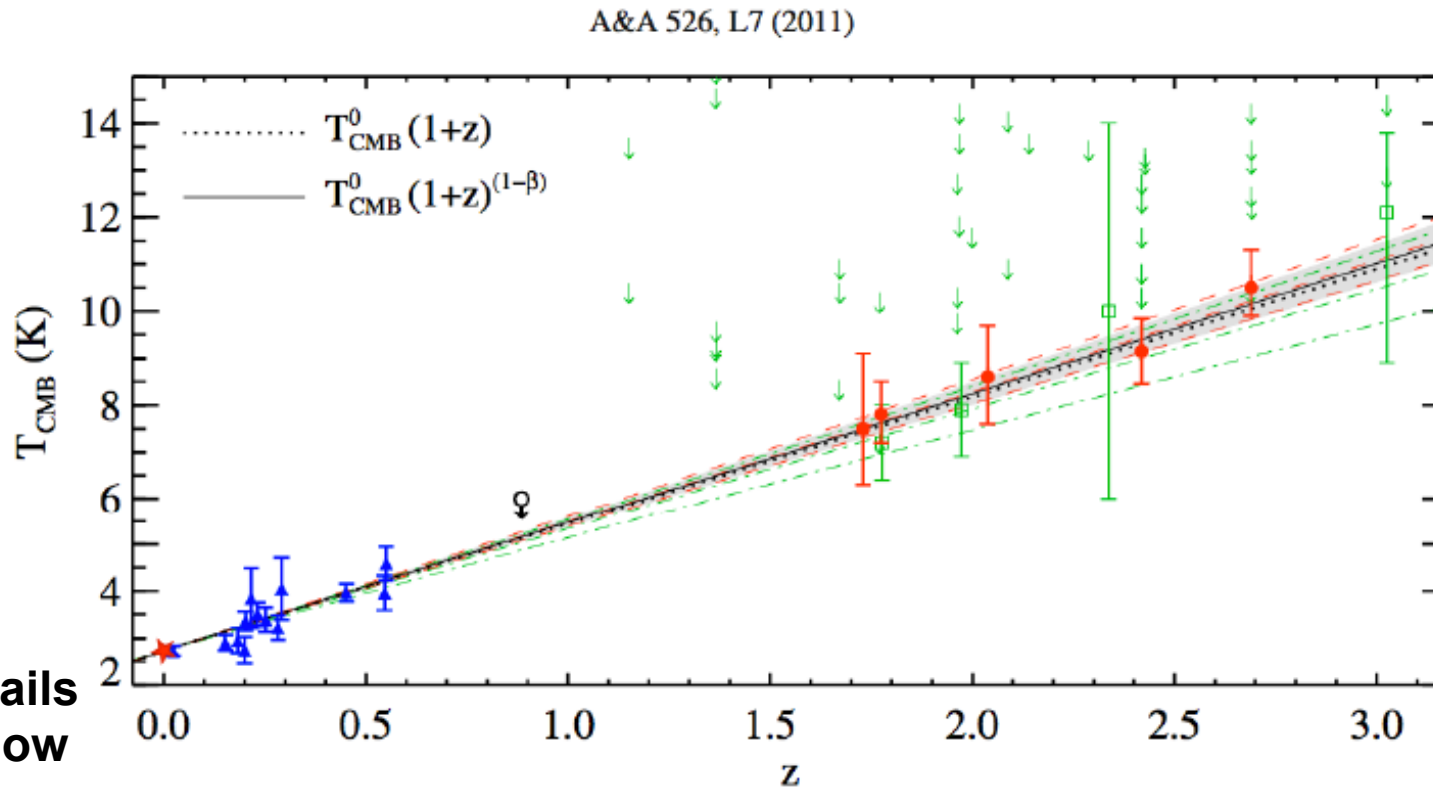
It tells us about the unseen early universe.

For example, matter / antimatter ratio $\sim 10^9$ today, all linked.

- › As $T \sim 1/a$, at earlier times, with smaller scale factor, the Universe would have been hotter – leads to the picture of a **Hot Big Bang**.
- › Currently the CMB photons do not interact significantly with matter.
- › As temperature increases back in time eventually the radiation will be hot enough to ionize the neutral atoms and a plasma will be formed.
- › In a plasma there is a large cross-section for interaction between the photons and electrons (i.e. Thompson scattering).
- › Electrons and photons maintain thermal equilibrium.



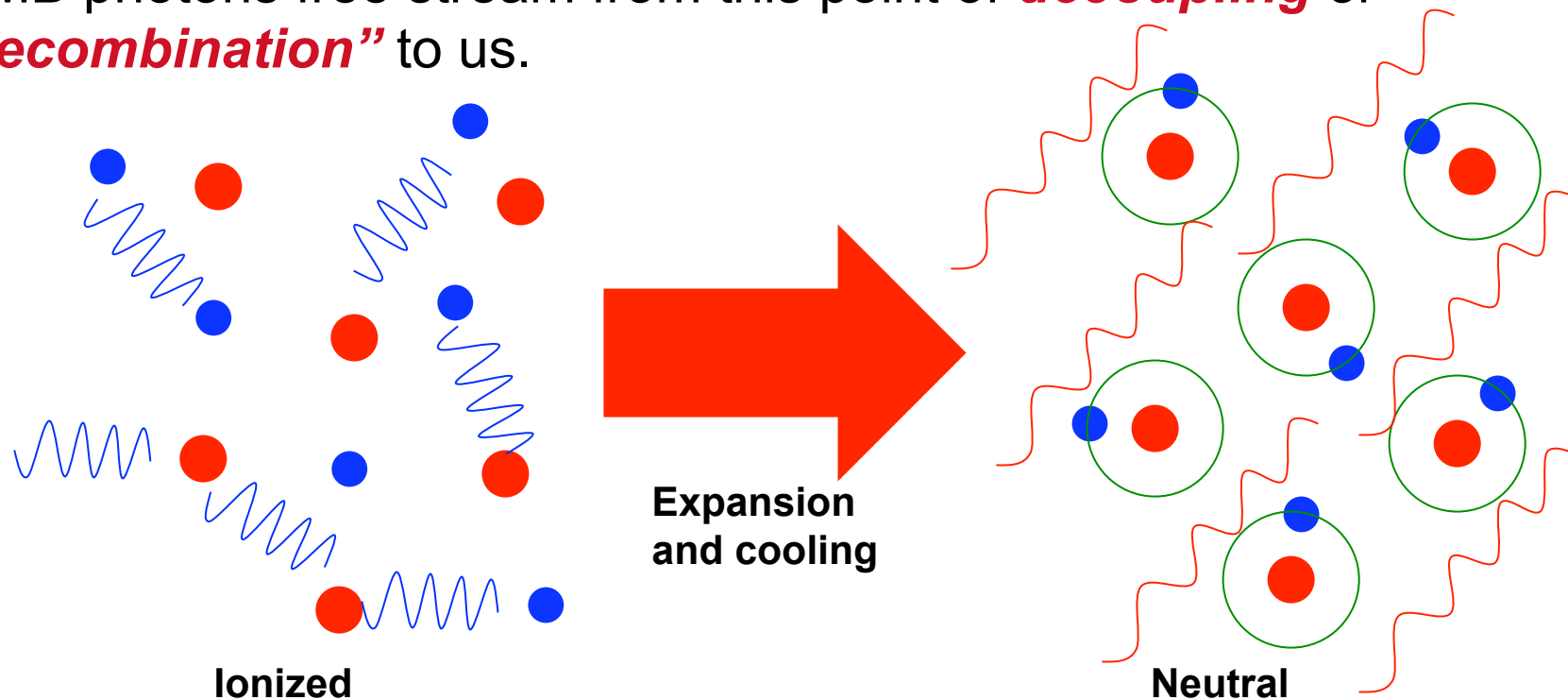
We actually measure the temperature rising with z – How about that?



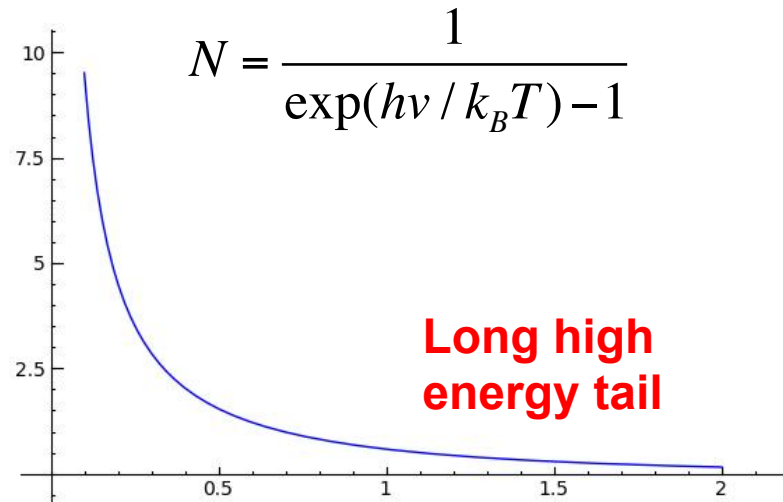
Ignore details
shown below

Fig. 4. Black-body temperature of the cosmic microwave background radiation as a function of redshift. The star represents the measurement at $z = 0$ (Mather et al. 1999). Our measurements based on the rotational excitation of CO molecules are represented by red filled circles at $1.7 < z < 2.7$. Other measurements at $z > 0$ are based (i) on the S-Z effect (blue triangles at $z < 0.6$, Luzzi et al. 2009) and (ii) on the analysis of the fine structure of atomic carbon (green open squares: $z = 1.8$, Cui et al. 2005; $z = 2.0$, Ge et al. 1997; $z = 2.3$, Srianand et al. 2000; $z = 3.0$, Molaro et al. 2002). Upper limits come from the analysis of atomic carbon (from the literature and our UVES sample, see Srianand et al. 2008) and from the analysis of molecular absorption lines in the lensing galaxy of PKS 1830-211 (open circle at $z = 0.9$, Wiklind & Combes 1996). The dotted line represents the adiabatic evolution of T_{CMB} as expected in standard hot Big-Bang models. The solid line with shadowed errors is the fit using all the data and the alternative scaling of $T_{\text{CMB}}(z)$ (Lima et al. 2000) yielding $\beta = -0.007 \pm 0.027$. The red dashed curve (resp. green dashed-dotted) represents the fit and errors using S-Z + CO measurements (resp. S-Z + atomic carbon).

- › As the Universe cools, the electrons and protons will eventually be able to form stable atoms.
- › The cross section for interactions between neutral atoms and photons is much lower.
- › CMB photons free stream from this point of **decoupling** or **“recombination”** to us.



- › When did decoupling occur? Photons have a Planck distribution:



$$N = \frac{1}{\exp(h\nu / k_B T) - 1}$$

$$F_\gamma(h\nu > E) \approx \int_{E/h}^{\infty} e^{(-E/k_B T)} d\nu \bigg/ \int_0^{\infty} e^{(-E/k_B T)} d\nu$$

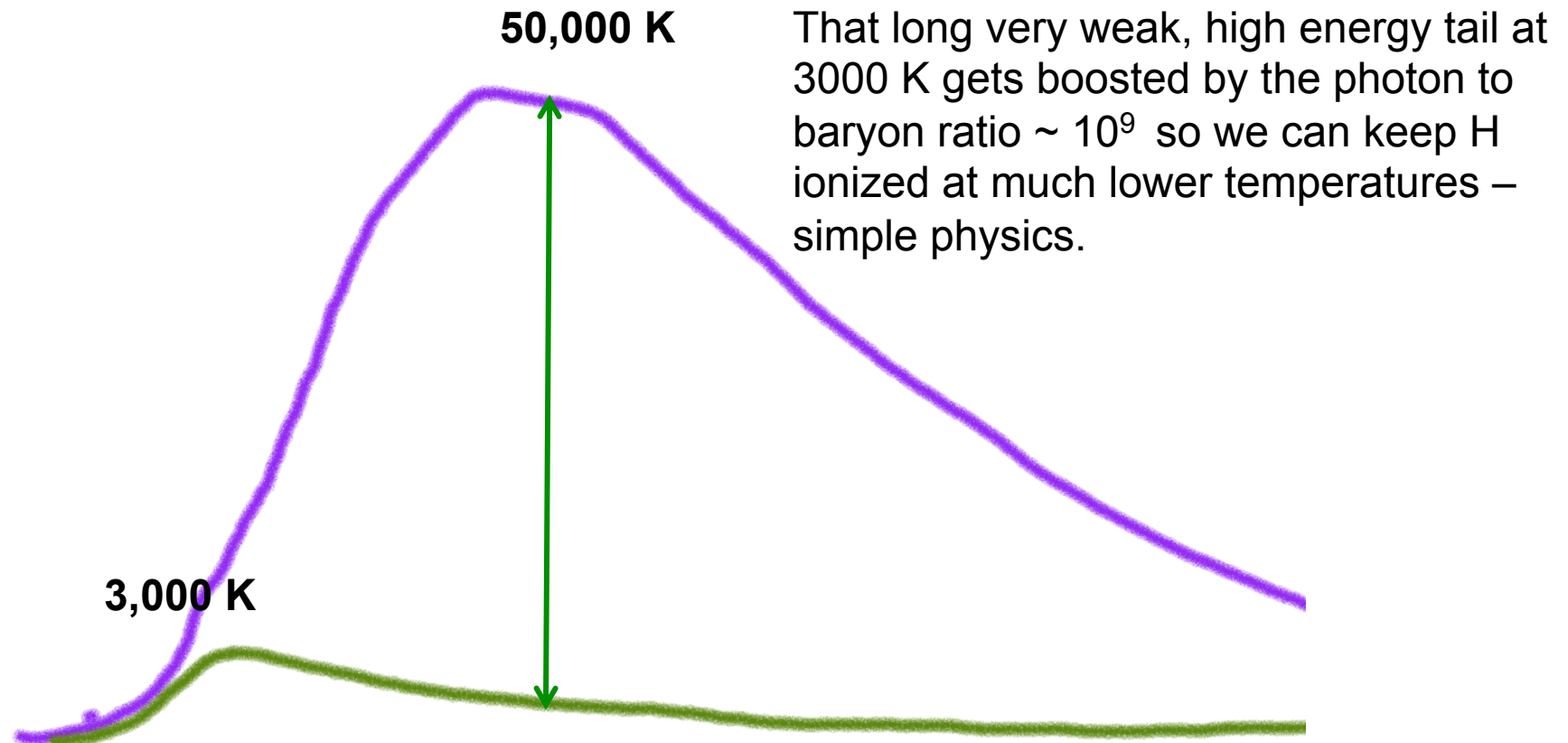
$$F_\gamma(h\nu > E) \approx e^{(-E/k_B T)}$$

$$\frac{E}{k_B T} \approx \ln\left(\frac{N_\gamma}{N_b}\right) \approx \ln(10^9) \approx 20$$

- › Need at least one photon per atom with energy higher than 13.6eV to maintain ionization.
- › The BB temperature for this energy is $T = \frac{13.6\text{eV}}{3k_B} \approx 50,000\text{K}$
- › But $\times 10^9$ more photons means that the tail of the BB distribution can maintain ionization. Detailed calculations give: $T \approx 3000\text{K}$
- › And this is what we expect since today's temperature is only about 3K.



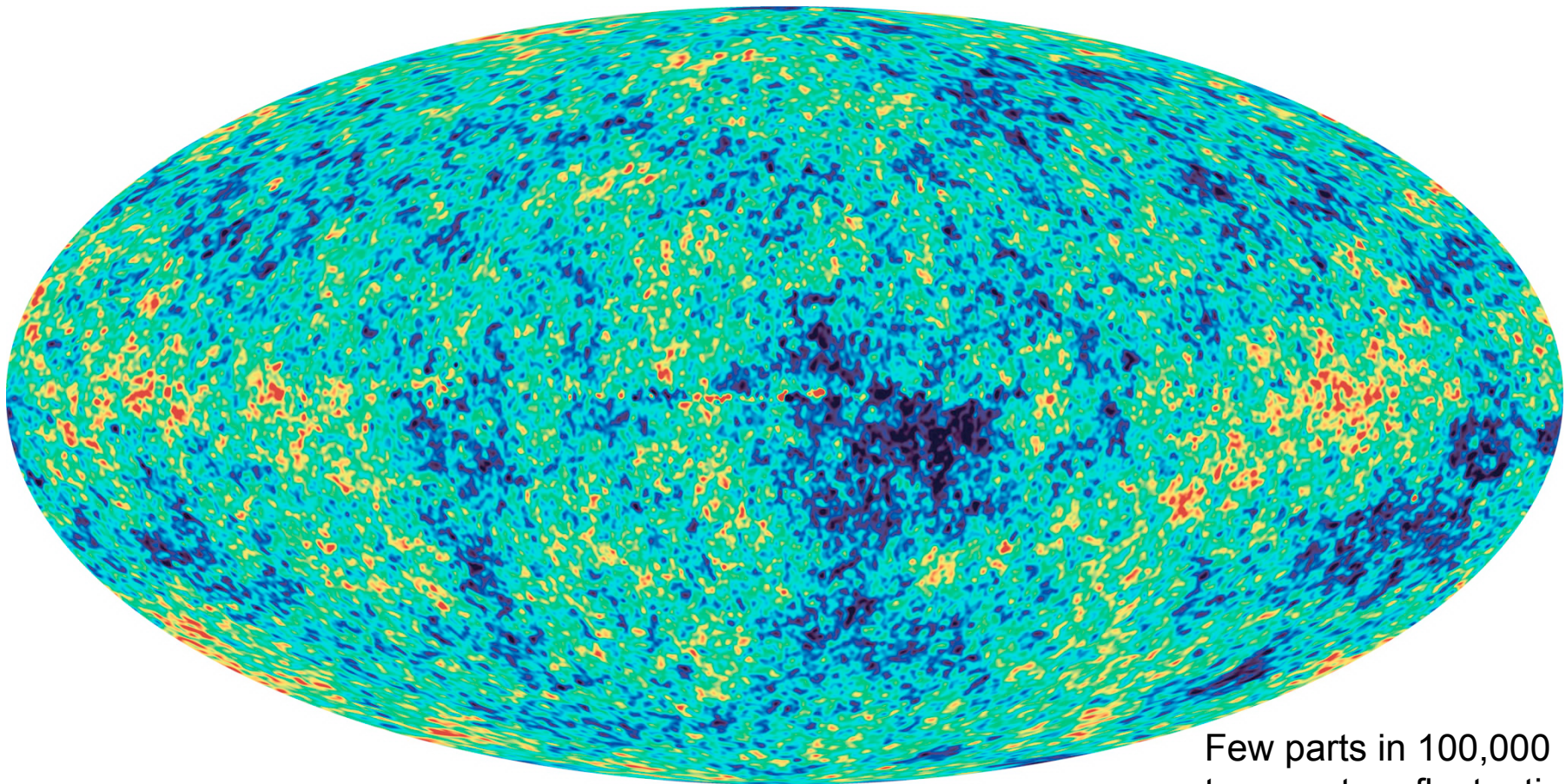
Why a lower temperature works for CMB





8.3 The CMB and horizons

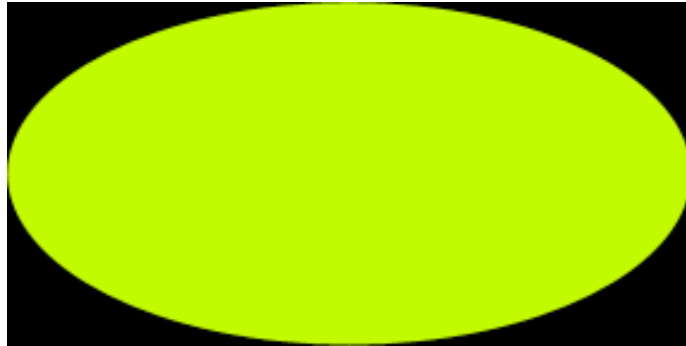
› Q: Why does the CMB look so uniform?



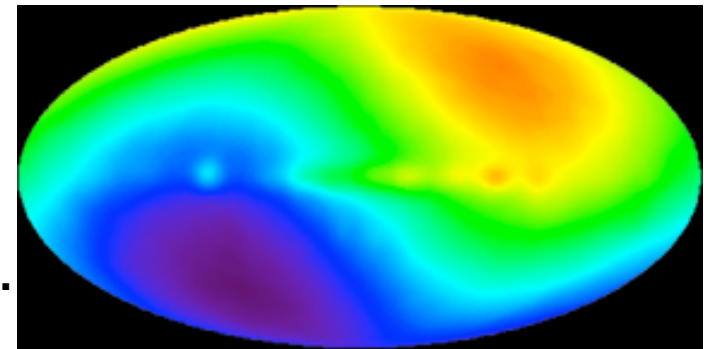
Few parts in 100,000
temperature fluctuations



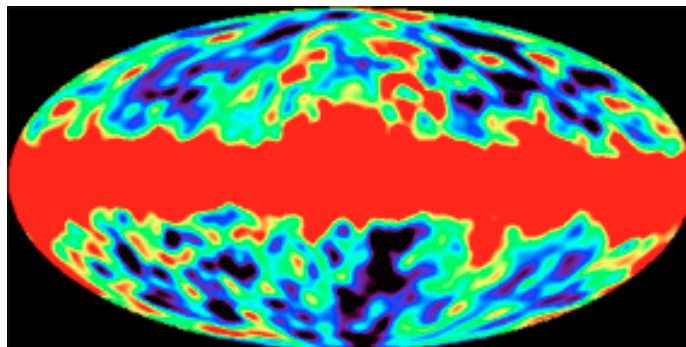
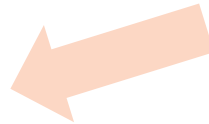
CMB and horizons



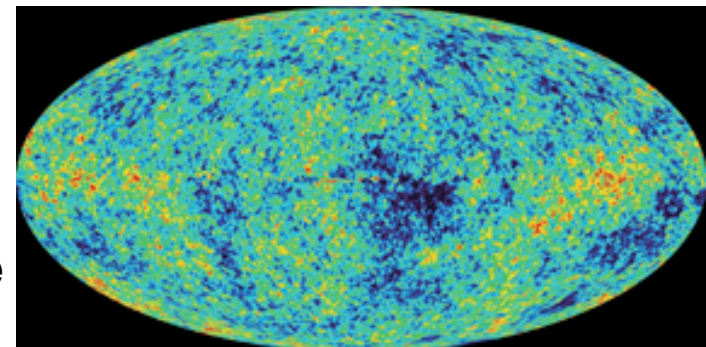
Simple view of the CMB (low contrast).



The CMB dipole (our motion in the Universe).



Milky Way structure in the microwave sky.



The CMB temperature fluctuations.

Images from WMAP.

- › Q: If the Universe has age t , what is the furthest distance we can see?

radial path of a photon:
$$\int_0^r \frac{dr}{[1 - kr^2]^{1/2}} = \int_0^t \frac{cdt}{a(t)}$$

- › Look at the solution for EdS case:

$$a(t) = a_0 \left(\frac{t}{t_0} \right)^{2/3}$$

Matter dominated

$$d_H(t) = ra(t) = a(t) \int_0^t \frac{cdt'}{a(t')} = a_0 \left(\frac{t}{t_0} \right)^{2/3} \int_0^t \frac{cdt}{a_0 (t/t_0)^{2/3}}$$

$$d_H(t) = ct^{2/3} \int_0^t \frac{dt}{t^{2/3}} = ct^{2/3} [3t^{1/3}] = \boxed{d_H(t) = 3ct}$$

- › This is the furthest **proper** (=comoving at t_0) distance that an event can currently be observed and is called the **Particle Horizon**.
- › In this case the scale factor, $a(t)$, increases at $t^{2/3}$, but the horizon increases faster.
- › The fraction of the Universe that is visible, **and in causal contact**, increases with time, and decreases towards the Big Bang.

- › Q: Why does the CMB look so uniform?
- › The particle horizon gives the maximum distance that a photon can propagate?

$$d_H(t) = 3ct \text{ for EdS Universe}$$

$$\text{then use } a(t) = a_0 \left(\frac{t}{t_0} \right)^{2/3} \quad \text{so } d_H(a) = 3ct_0 \left(\frac{a}{a_0} \right)^{3/2}$$

$$\text{and recall } t_0 = \frac{2}{3H_0} \quad \text{so}$$

$$d_H(z) = \frac{2c}{H_0} \frac{1}{(1+z)^{3/2}}$$

This horizon distance (proper distance) at $z=1100$ was not very large, in fact less than ~ 200 kpc. You need to convert $H_0 = 70$ km/s/Mpc to SI units to see that trivially, and recall that 1 parsec $= 3 \times 10^{16}$ m.

That's a smaller sphere than the outer halo of the Milky Way today.

The CMB and horizons

- Now we have the particle horizon at redshift z , we can also calculate the angular size of the horizon (recalling the angular diameter distance, d_A):

$$\theta = \frac{d_H(z)}{d_A(z)} = \frac{\text{(proper horizon distance at } z\text{)}}{\text{(angular diameter distance to } z\text{)}}$$

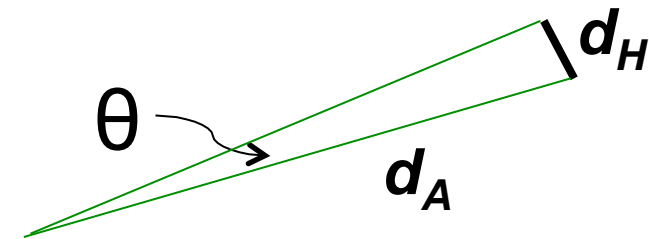
For an EdS Universe :

$$d_A(z) = \frac{d_c(z)}{(1+z)} = \frac{2c}{H_0} \left[1 - \frac{1}{(1+z)^{1/2}} \right] \frac{1}{(1+z)} \quad \text{and} \quad d_H(z) = \frac{2c}{H_0} \frac{1}{(1+z)^{3/2}}$$

$$\text{So } \theta = \frac{1}{(1+z)^{1/2}} \left[\frac{1}{1 - 1/(1+z)^{1/2}} \right]$$

CMB decoupling is at $z = 1100$, so

$$\theta = 0.03 \text{ radians} = 1.7 \text{ degrees}$$



- Areas of the CMB separated by more than ~ 1.7 deg have not been in causal contact. **Why do they look the same?**



WMAP CMB measurements

- › Smooth to 1 part in $\sim 100,000$ – but how?

How big is 2°
on the full sky?

