PHYS 2013, 2913, 2923
Astrophysics & Cosmology

Lectures 8-10

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13 Lecture series: first 7 given by Celine
6 Lectures: Wednesday, Thursday (Zoom)
3 Tutorials: Tuesday (Zoom)
1 Assignment, 2 Quizzes, 1 Exam
30 Oct – 20 Nov, 2020

Signposts – so where are we going with the lectures?

Celine introduced important ideas in modern cosmology, e.g. Friedmann equations, using Chapters 1-5, and some of Chapters 6-7, of Liddle's book. I will now introduce the entire Universal narrative – from inflation to galaxies (and stars) – without getting bogged down in advanced GR/QM concepts. For those that stay the course, Archil and I revisit these ideas in depth in PHYS 4122 two years from now. I now use the more interesting second half of Liddle's book (with parts of Ryden's book) starting with the transition chapters 6 and 7.

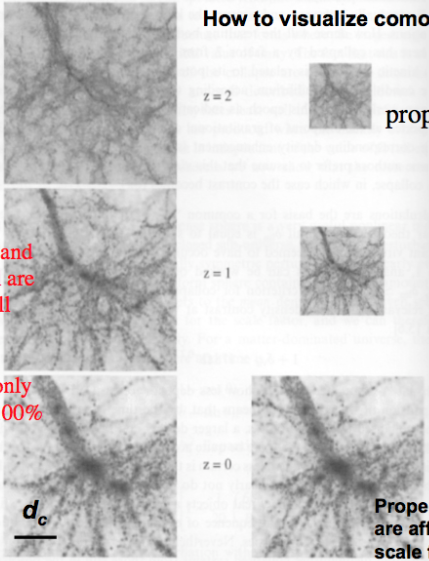
- Lectures 8-9 are about the main cosmological parameters and equations driving evolution, and the complex vacuum processes that get us from the Big Bang to a power spectrum of fluctuations, $P(k)$.
- The remaining lectures are about how we got from the first elements, the cosmic microwave background ($\delta \lesssim 10^{-5}$), and the first stars, to present day galaxies ($\delta \gtrsim 10^{2}$).
- Lectures 9-10 are about Big Bang nucleosynthesis, the creation of the first elements, and the Universal fog that came to be.
- Lectures 10-11 are about the cosmic microwave background and the onset of structure formation driven by dark matter.
- Lectures 12-13 are about the first stars, the formation of heavy elements, the first black holes, and the epoch of reionization when the fog lifted. We talk about how gas falls into galaxies and gets processed, and how galaxies evolve.

Proper and comoving coordinates

How to visualize comoving coords.

comoving coords

proper coords



in co-moving coords, distribution of matter and gravitational potential are mostly frozen in for all time

non-linear evolution only increases by at most 100%

Comoving distances do not change with time, expansion corrected

Proper distances are affected by the scale factor

Hubble parameter & Hubble constant

There's a linear relation between recession speed v and **proper distance** d,

$$v(t) = H(t) \cdot d(t) \tag{1}$$

where $H(t) = \dot{a}/a$ is the **Hubble parameter**, i.e. for all universes in a FRW metric,

$$H(t)^2 = \frac{8\pi G}{3c^2}\varepsilon(t) - \frac{\kappa c^2}{R_0^2}\frac{1}{a(t)^2} \tag{2}$$

At the present time (t_0, subscript zero important), we define the **Hubble constant**

$$H_0 = H(t_0) = \left(\frac{\dot{a}}{a}\right)_{t=t_0} = 70 \pm 7 \text{ km s}^{-1}\text{ Mpc}^{-1} \tag{3}$$

The Friedmann equation evaluated today,

$$H_0^2 = \frac{8\pi G}{3c^2}\varepsilon_0 - \frac{\kappa c^2}{R_0^2} \tag{4}$$

H_0, ε_0, R_0 and κ are all things we can measure, and this is still a major industry in cosmology. How do we do that? Just a few examples.

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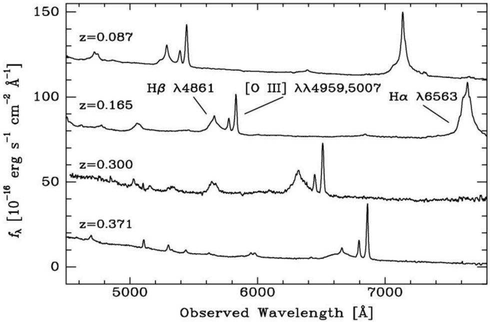
Measurable 1: cosmological redshift, z

Using **spectroscopy**, spectra are easy to obtain (flux density f_λ vs. wavelength λ) and spectral emission or absorption lines have known wavelengths in the lab (at rest). The observed wavelengths are "redshifted" by cosmic expansion and we can convert that to a "recession velocity" v at low redshift (no need for SR/GR corrections), i.e. $v = cz$ where $z \ll 1$.



Two redshift estimates from two measured lines λ_A and λ_B that are known calibrators λ_0 and λ_1 in same spectrum:

$$z_{A,0} = \frac{\lambda_A - \lambda_0}{\lambda_0}$$

$$z_{B,1} = \frac{\lambda_B - \lambda_1}{\lambda_1}$$

The observed ratio λ_A/λ_B and the lab ratio λ_0/λ_1 are unchanged by z within measurement errors. This helps to identify lines in crappy spectra, a common problem.

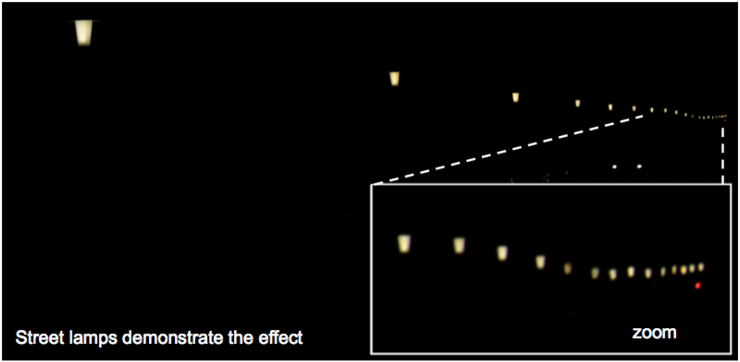
- Identify lines via wavelength ratios, which are unchanged by redshift

Measurable 2: source flux $f \rightarrow$ luminosity distance d_L

Using **photometry**, a source flux is easy to obtain, and these can be converted to a luminosity distance, d_L, if the source has a *known* intrinsic luminosity, L.

If the Universe is kind enough to create celestial sources that are like L = 100W light bulbs, then by measuring their apparent brightness f, we can measure their luminosity distances accurately, independent of the assumed cosmology. We call this photometry, a big field of astronomy. (We work in weird units called magnitudes!)

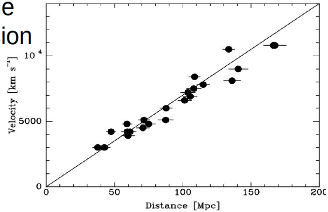
Basically $d_L = \sqrt{(L / 4 \pi f)}$

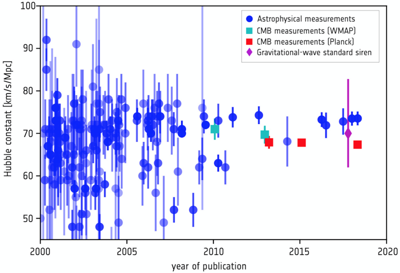


Hubble diagram

We bring these measurables together (LHS plot). The velocities v were measured from the redshift. The distances d were measured from the luminosity distance d_L. The Hubble constant is simply the gradient since $v = cz = H_0 d$. Note: some of the error is real due to the peculiar motions we talked about, some is measurement error.

Hubble Expansion

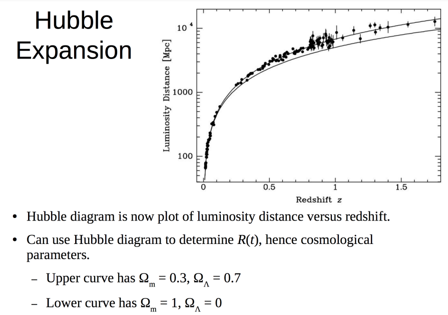


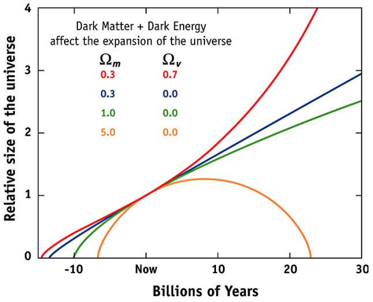


The RHS plot shows how H_0 estimates have evolved with human time, and there is still a bit of controversy (tension).

Hubble diagram, supernovae & dark energy

The LHS plot shows modern data from supernovae (Type Ia) that are the best distance calibrators we know. The total matter-energy budget (Ω) was found to be split between **dark matter + baryons** ($\Omega_m = 0.3$) and **dark energy** ($\Omega_\Lambda = 0.7$). Brian Schmidt and friends were awarded the 2011 Nobel Prize in Physics.





The RHS plot shows the most likely Universe (red); pure matter (blue, green, orange) models & pure energy model (not shown; sits above red curve), all completely ruled out. **The age of the Universe is 13.8 billion years.**

Empty Universe (not shown): $\varepsilon_0 = 0$ now, such that $H_0^2 = -\kappa c^2/R_0^2 \rightarrow$ minimum radius of curvature (size) of $R_0 = c/H_0 \approx 4$ Gpc, smaller than the observable Universe.

Critical density today: $\varepsilon_{c,0}, \rho_{c,0}, \Omega_0$

In a flat Universe ($\kappa \approx 0$, $\Omega_0 = \Omega_\Lambda + \Omega_m \approx 1$), we can define a critical energy density ε_c

$$H(t)^2 = \frac{8\pi G}{3c^2}\varepsilon(t) \quad (5)$$

$$\varepsilon_c(t) = \frac{3c^2}{8\pi G}H(t)^2 \quad (6)$$

If you infer $\varepsilon_{c,0}$ is higher or lower today, then $\kappa \neq 0$. Since we think $\kappa \approx 0$, what is that critical energy density?

$$\varepsilon_{c,0} = \frac{3c^2}{8\pi G}H_0^2 = (8.3 \pm 1.7) \times 10^{-10} \text{ J m}^{-3} \quad (7)$$

$$\rho_{c,0} = \varepsilon_{c,0}/c^2 = (9.2 \pm 1.8) \times 10^{-27} \text{ kg m}^{-3} \quad (8)$$

or one H atom per 200 L, or 140 $M_\odot$ per kpc^3; most locked up in the **dark sector**.

The dimensionless **density parameter** gives a variant on the Friedmann equation:

$$\Omega(t) \equiv \frac{\varepsilon(t)}{\varepsilon_c(t)} \quad \rightarrow \quad 1 - \Omega(t) = -\frac{\kappa c^2}{R_0^2 a(t)^2 H(t)^2} \quad (9)$$

such that today ($\Omega_0 = 1$ is a flat Universe)

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Some light, mostly dark

Fluid equation

The Friedmann equation depends on two unknowns: $a(t)$ and $\varepsilon(t)$ (or equivalently $H(t)$ and $\Omega(t)$). Consider a sphere with co-moving radius r_s expanding with the Universe, so that its proper radius is $R_s(t) = a(t)r_s$. Thus

$$V(t) = \frac{4\pi}{3} r_s^3 a(t)^3 \tag{11}$$

$$\dot{V} = \frac{4\pi}{3} r_s^3 (3a^2\dot{a}) = V(3\frac{\dot{a}}{a}) \tag{12}$$

For the internal energy E

$$E(t) = V(t)\varepsilon(t) \tag{13}$$

$$\dot{E} = V\dot{\varepsilon} + \dot{V}\varepsilon = V(\dot{\varepsilon} + 3\frac{\dot{a}}{a}\varepsilon) \tag{14}$$

Like the Friedmann equation, the **first law of thermodynamics** ($dQ = dE + PdV$) is another energy conservation equation. Since the expanding Universe is adiabatic, $dQ = 0$, thus entropy $dS = dQ/T = 0$. Thus $\dot{E} + P\dot{V} = 0$, and

$$V(\dot{\varepsilon} + 3\frac{\dot{a}}{a}\varepsilon + 3\frac{\dot{a}}{a}P) = 0 \tag{15}$$

$$\dot{\varepsilon} + 3\frac{\dot{a}}{a}(\varepsilon + P) = 0 \qquad \textbf{fluid equation} \tag{16}$$

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Acceleration equation and equations of state

If we use the $\dot{a}/a$ form of the Friedmann equation, it's trivial to combine this with the fluid equation to get the **acceleration equation** (note P and ε have the same units),

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2}(\varepsilon + 3P) \tag{17}$$

Many papers that examine the state of the Universe solve for all three equations concurrently - these are **equations of state**. Stuff like atoms, photons, dark matter particles, etc. contribute positive energy density (=*positive pressure*), that GR tells us through the energy-momentum (stress-energy) tensor **slows the Universe down.** (Don't **ever** think of a balloon in 3D being pushed by gas or radiation pressure; that's ok for local dynamics but not for cosmic dynamics for Universe as a whole; we need GR for that.)

So how can the Universe be expanding at all, let alone accelerating?

It must all come down to a mysterious property of space called the **vacuum**, more specifically its **non-zero energy state** (QM).

Vacuum energy

Einstein knew Poisson's equation $\nabla^2\Phi = 4\pi G\rho$ ($\rho \neq 0$) implies (i) a static universe must collapse; (ii) an initially expanding Universe must expand forever, or turn around then collapse.

He wanted to make the Universe unchanging since it looked that way in 1920. He balanced the equation with

$$\nabla^2\Phi + \Lambda = 4\pi G\rho \tag{18}$$

(static universe) using a **cosmological constant** Λ, something he later regretted because it's unstable. *But he was on the right track!*

By definition, Λ **must** supply **gravitational repulsion** as in the equation, i.e. constant **negative pressure**. How to understand that?

You have experienced that, maybe, e.g. evacuating air from a container (it implodes); breathing back in is positive pressure.

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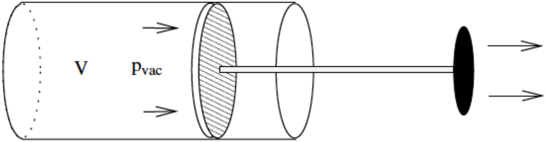
Negative pressure **caused by** the expansion

When you pull back on a bicycle pump, it's hard work if you close the hole at the end. The energy you create/exert by pulling on the handle is $\varepsilon_{\rm vac}.dV$. The negative pressure is **caused by** the exertion. The work done due to the vacuum is $p_{\rm vac}.dV$, thus $p_{\rm vac} = -\varepsilon_{\rm vac}$.



We need a relation like $P = P(\varepsilon)$ to close the system of equations above, the last equation of state. We write (for dimensionless **w parameter**)

$$P = w \cdot \varepsilon \tag{19}$$

The perfect gas law is one such relation ($3kT = \mu\langle v^2\rangle$, μ = mean particle mass)

$$P_{\rm non-rel} = \frac{\rho}{\mu}kT \approx \frac{kT}{\mu c^2}\varepsilon \tag{20}$$

$$= w \cdot \varepsilon_{\rm non-rel} \tag{21}$$

where $w \approx \langle v^2\rangle/3c^2 \ll 1$ ($w \sim 10^{-12}$ for N_2 at room temperature).

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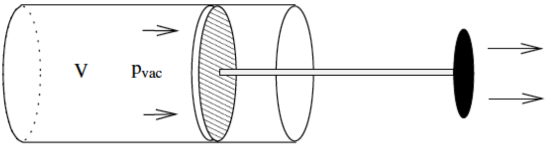

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Cosmological constant Λ

For photons and other massless particles ($\langle v^2 \rangle \sim c^2$)

$$P_{\rm rel} = \frac{1}{3}\varepsilon_{\rm rel} \tag{22}$$

Returning to the Friedmann equations,

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}\varepsilon - \frac{\kappa c^2}{R_0^2 a^2} + \frac{\Lambda}{3} \tag{23}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2}(\varepsilon + 3P) + \frac{\Lambda}{3} \tag{24}$$

From the first equation, the Λ equation is adding a new component with energy density $\varepsilon_\Lambda = \frac{c^2}{8\pi G}\Lambda$. To have ε_Λ constant with time,

$$P_\Lambda = -\varepsilon_\Lambda = -\frac{c^2}{8\pi G}\Lambda \tag{25}$$

i.e. constant pressure and $w = -1$.

Vacuum energy – are Λ & dark energy related?

Vacuum energy is a quantum phenomenon, nothing to do with cosmic dynamics, emerging out of the vacuum due to Heisenberg's Uncertainty Principle, i.e. $\Delta E \cdot \Delta t \gtrsim \hbar (= \frac{h}{2\pi})$.

Vacuum energy is used all over physics, and has been demonstrated many times:

- Renormalization theory
- Casimir effect
- Lamb shift
- Einstein's spontaneous emission
- Positron-electron pairs
- Hawking radiation
- Cosmic inflation

It may also be related to Λ and dark energy, *but* QM and GR disagree by 10^{120} on what it should be – **the single biggest failing in all of physics.**

Universal acceleration observed today – emerging from the cosmic vacuum around $z \sim 0.5$ – has happened before, i.e. the epoch of **cosmic inflation** just after the Big Bang. The QM prediction for inflationary $\varepsilon_{\rm vac}$ seems about right, 10^{120} bigger than observed $\Lambda \sim 10^{-9}$ J m^{-3} today. So just what is dark energy? Weird.

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What is this strange vacuum energy ?

How can a vacuum **release** energy as it expands, fundamental to inflation, to Λ?

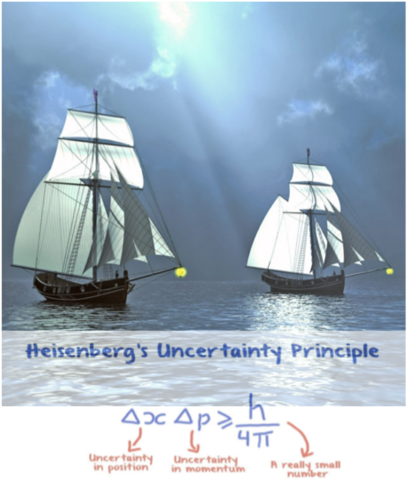
The vacuum is **not** empty – it's like a bank balance that averages to zero, but this hides a lot of credits and debits.

QM does not allow **nothing** to exist. Virtual particles come and go all the time as long as they obey H.U.P.

The vacuum has **potential energy** that can be released through a phase transition. **Space** has a substance which is why we can bend it in **Einstein's general theory of relativity**.

How do we know?

Here's the most elegant demo: boats on the ocean always drift together if under the force of the waves. Why? **Casimir effect.**



Cosmic inflation (Guth 1981)

We start with the acceleration equation,

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2}(\varepsilon + 3P) \tag{26}$$

For outward acceleration, we need $\ddot{a} > 0$ and $P < -\varepsilon/3$, equivalently $w < -1/3$. Adopt a $+$ve cosmological constant Λ_i ($w = -1$), in an era when it dominated the Universe,

$$\frac{\ddot{a}}{a} = \frac{\Lambda_i}{3} > 0 \qquad \text{(acceleration)} \tag{27}$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{\Lambda_i}{3} > 0 \qquad \text{(Friedmann)} \tag{28}$$

There is a functional form that fits both of these requirements to describe the **inflation epoch**,

$$a(t) \propto e^{H_i t} \qquad \text{where } H_i = \sqrt{\frac{\Lambda_i}{3}} \tag{29}$$

Note the inflationary Hubble constant H_i really is a constant. We somehow need inflation to switch on ($t = t_i$), transition to our Universe today, and switch off ($t = t_f$). This is not easy to do.

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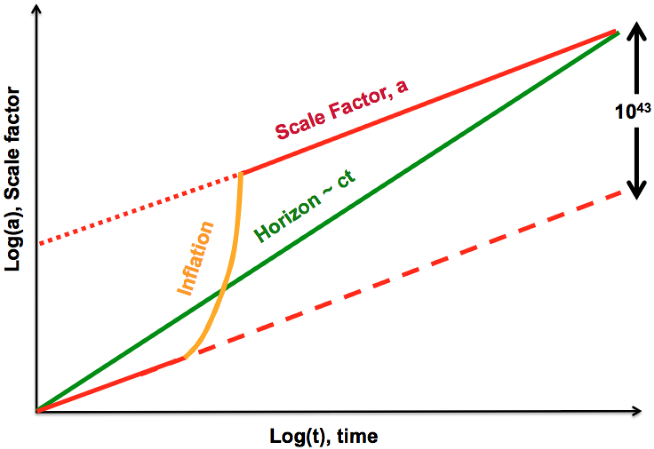
Inflation: when is enough enough?



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Inflation: space-time expanded faster than light

How can the Universe expand faster than the speed of light during inflation? (Advanced)

You ask a good question, one whose answer lies in the subtle difference between *expansion* that is faster than the speed of light and the *propagation of information* that is faster than the speed of light. The latter is forbidden by fundamental physical laws, but the former is allowed; that is, as long as you are not transmitting any *information* (like a light pulse), you can make something happen at a speed that is faster than that of light. The expansion of the Universe is a "growth" of the spacetime itself; this spacetime may move faster than the speed of light relative to some other location, as long as the two locations can't communicate with each other (or, in terms of light rays).

Cosmic inflation: a simple model

The early Universe was driven by a radiation-dominated phase ($a(t) \propto t^{1/2}$), so let's insert an inflationary phase into that (matter-dominated, when $a(t) \propto t^{2/3}$, came much later before the rise of Λ_0),

$$a(t) = a_i(t/t_i)^{1/2} \qquad t < t_i \tag{30}$$

$$= a_i e^{H_i(t-t_i)} \qquad t_i < t < t_f \tag{31}$$

$$= a_i e^{H_i(t_f-t_i)}(t/t_f)^{1/2} \qquad t > t_f \tag{32}$$

Over the timeframe $t_i < t < t_f$, the scale factor increased by

$$\frac{a(t_f)}{a(t_i)} = e^N \qquad \text{where } N \equiv H_i(t_f - t_i) \tag{33}$$

If the inflationary phase $t_f - t_i$ was long compared to the Hubble time H_i^{-1}, the number of e-folds N could be enormous. Let's take Alan Guth's GUT model lasting for $N \sim 100$ Hubble times, $t_i \approx t_{\rm GUT} \approx 10^{-36}$ s, $t_f \approx 10^{-34}$ s. Here $H_i \approx t_{\rm GUT}^{-1} \approx 10^{36}$ s^{-1},

$$\frac{a(t_f)}{a(t_i)} \sim e^{100} \sim 10^{43} \tag{34}$$

Note that Λ_i was far larger than our Λ_0 today that has $\varepsilon_{\Lambda,0} \approx 0.7\varepsilon_{c,0} \approx 0.004$ TeV m^{-3}.

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$$\begin{aligned} a(t) &= a_i(t/t_i)^{1/2} && t < t_i && (30) \\ &= a_i e^{H_i(t-t_i)} && t_i < t < t_f && (31) \\ &= a_i e^{H_i(t_f-t_i)}(t/t_f)^{1/2} && t > t_f && (32) \end{aligned}$$

Over the timeframe $t_i < t < t_f$, the scale factor increased by

$$\frac{a(t_f)}{a(t_i)} = e^N \qquad \text{where } N \equiv H_i(t_f - t_i) \tag{33}$$

If the inflationary phase $t_f - t_i$ was long compared to the Hubble time H_i^{-1}, the number of e-folds N could be enormous. Let's take Alan Guth's GUT model lasting for $N \sim 100$ Hubble times, $t_i \approx t_{\text{GUT}} \approx 10^{-36}$ s, $t_f \approx 10^{-34}$ s. Here $H_i \approx t_{\text{GUT}}^{-1} \approx 10^{36}$ s^{-1},

$$\frac{a(t_f)}{a(t_i)} \sim e^{100} \sim 10^{43} \tag{34}$$

Note that Λ_i was far larger than our Λ_0 today that has $\varepsilon_{\Lambda,0} \approx 0.7\varepsilon_{c,0} \approx 0.004$ TeV m^{-3}.

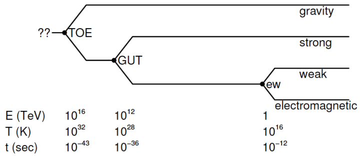
Cosmic inflation: a simple model, contd.

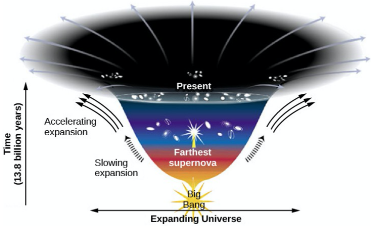
To produce exponential expansion with $H_i \approx 10^{36}$ s^{-1}, we would need

$$\varepsilon_{\Lambda_i} = \frac{c^2}{8\pi G}\Lambda_i = \frac{3c^2}{8\pi G}H_i^2 \sim 10^{105}\ \mathrm{TeV\ m^{-3}} \qquad (35)$$

over 100 orders of magnitude larger! Inflation is perceived as a great success because it solves the **flatness problem**, the **monopole problem**, and the **horizon problem**. Any non-uniformity would be stretched hugely.

It's all kind of crazy. At $t = t_f$, our currently observable Universe was crammed into a sphere with proper radius $\sim$ 90 cm. At $t = t_i$, this same Universe was crammed into a sphere with proper radius $\sim 3 \times 10^{-44}$ m. The observable horizon at $t = t_i$ was far larger, i.e. $ct_i = 3 \times 10^{-28}$ m. *But the Universe today is far larger than the observable horizon.*



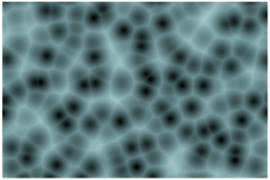


Planck units lie at the heart of quantum cosmology – this is where fluctuations come from

- What existed around the Big Bang was a space-time **quantum foam** where quantum mechanics dominated over everything, we think
- The smallest time unit was the **Planck time**,

or $5\text{x}10^{-44}$ sec, thus **Planck length**, $1.6\text{x}10^{-35}$ m.

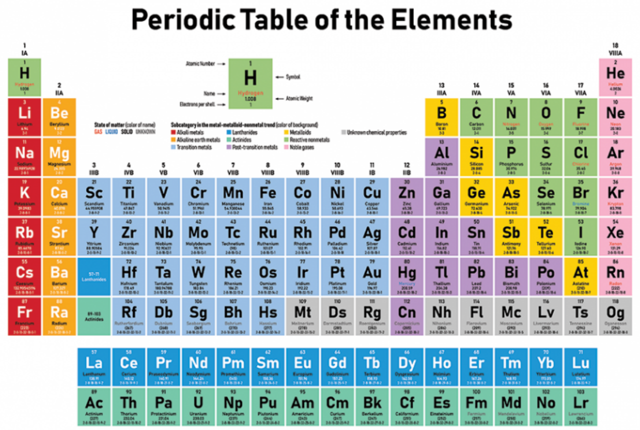
This is the natural radiated wavelength, thus **Planck temperature**, $1.4\text{x}10^{32}$ K, thus **Planck energy** and **Planck mass** follow.



Name	Dimension	Expression	Value (SI unit)s[2]
Planck length	Length (L)	$l_P = \sqrt{\frac{\hbar G}{c^3}}$	$1.616\,255(18) \times 10^{-35}$ m[6]
Planck mass	Mass (M)	$m_P = \sqrt{\frac{\hbar c}{G}}$	$2.176\,435(24) \times 10^{-8}$ kg[7]
Planck time	Time (T)	$t_P = \frac{l_P}{c} = \frac{\hbar}{m_P c^2} = \sqrt{\frac{\hbar G}{c^5}}$	$5.391\,245(60) \times 10^{-44}$ s[8]
Planck charge	Electric charge (Q)	$q_P = \sqrt{4\pi\varepsilon_0 \hbar c} = \frac{e}{\sqrt{\alpha}}$	$1.875\,545\,956(41) \times 10^{-18}$ C[9][4][10]
Planck temperature	Temperature (Θ)	$T_P = \frac{m_P c^2}{k_B} = \sqrt{\frac{\hbar c^5}{G k_B^2}}$	$1.416\,785(16) \times 10^{32}$ K[11]

Big Bang Nucleosynthesis: where do the elements come from?

The unimaginably hot Big Bang, followed by rapid cooling in the expansion, created the ideal environment for particle creation, particle fusion, element creation. The initial fundamental particles or building blocks (e.g. quarks, neutrinos, photons, electrons) emerged from vacuum fluctuations but, once formed, subsequent particles (like protons, neutrons) were fused together from the building blocks. How does this work?



Big Bang Nucleosynthesis: change in mean energy per photon with cosmic time, t

Recall that the Early Universe was radiation-dominated; the scale factor evolved as $a(t) \propto t^{1/2}$. The Universal temperature declined as $T(t) \propto 1/a$, thus we can write

$$T(t) \approx 10^{10}\ \mathrm{K} \left(\frac{t}{1\ \mathrm{second}}\right)^{-1/2} \tag{36}$$

$$kT(t) \approx 1\ \mathrm{MeV} \left(\frac{t}{1\ \mathrm{second}}\right)^{-1/2} \tag{37}$$

$$3kT(t) \approx 3\ \mathrm{MeV} \left(\frac{t}{1\ \mathrm{second}}\right)^{-1/2} \tag{38}$$

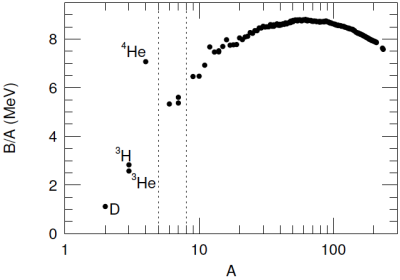
where the last equation is roughly the *mean energy per photon* at that time.

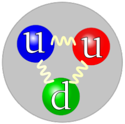
In Tuesday's tutorial, we look at some consequences with elementary calculations using my favourite online Astrophysics calculator (units!).
What was the mean energy at the Planck time and what is it today? When was the Universe operating like a Tevatron or Pevatron accelerator? What was special about The First Three Minutes, the title of Weinberg's book? For example, the rest mass energy of an electron is $E_e = m_e c^2 = 511$ MeV, so can we find a rough estimate of when electrons first emerged?

Big Bang Nucleosynthesis: binding energy per nucleon, B/A

Binding energy per nucleon is an important concept across all of physics and chemistry. If the environment exceeds this energy, the particle is split apart. So clearly, for a particle to survive, the mean energy per photon or other particle flying around needs to fall below B/A.

This simple fact sets up critical epochs when the all the different particles emerged, with the most fundamental particles emerging first.





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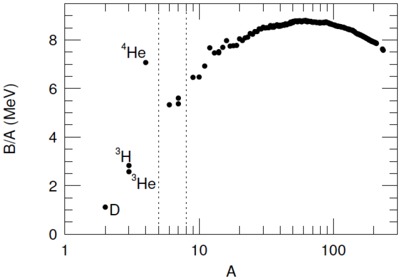
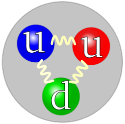
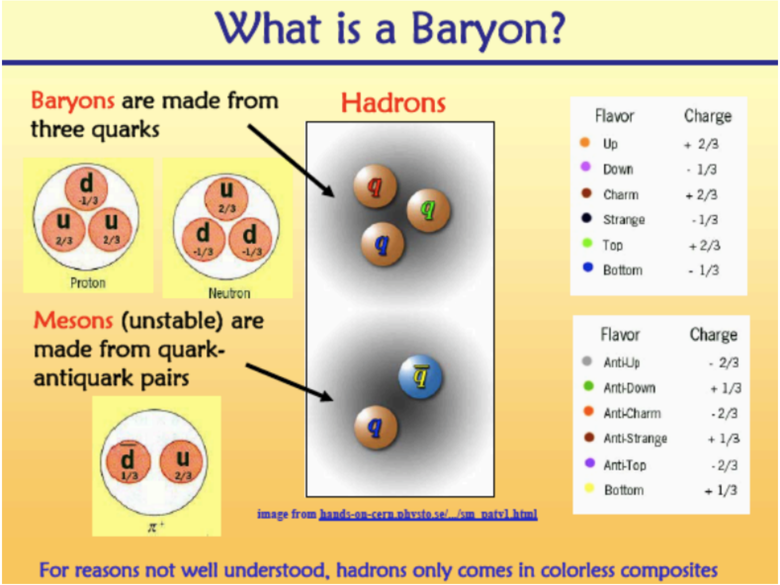




For example, quarks appeared at the **end** of the **quark epoch** when force laws emerged ($10^{-12} < t < 10^{-6}$ sec). After this, $\langle E \rangle$ dropped below $\sim$1000 MeV, i.e. binding energy of a proton (940 MeV).

Quark rest mass is only a few MeV, gluons have zero rest mass; 99% of proton mass is from relativistic motions and gluon forces within the proton. There followed free protons (live forever) and neutrons (15 minutes of fame). The latter is unstable to decay: $\boldsymbol{n} \rightarrow p + e^- + \bar{\nu}_e$

Astronomers speak of normal matter = baryons



Leptogenesis, Baryogenesis: asymmetry in the Universe

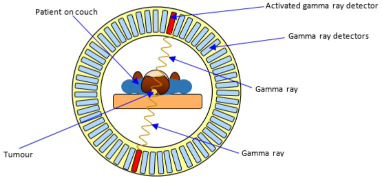
The Universe began with pure (radiative) energy, and devolved into energy and matter as the Universe expanded. At t = 0.1 seconds, $T = 3 \times 10^9$ K, $\langle E \rangle \approx$ 10 MeV, much greater than electron rest mass.

All virtual particles came as matter-antimatter pairs, or photon pairs, e.g. two gamma ray photons collide to produce an electron and a positron, and vice versa:

$\gamma + \gamma \rightleftharpoons e^- + e^+$

This was not a good time for electrons to survive for long.

For some amazing reason, this relationship is/was very slightly asymmetric because today, matter/antimatter $\sim 10^9$. **Where did all the antimatter go ??**



Positron emission tomography (PET) scanner

This is mass-energy equivalence ($E = mc^2$) and the underlying principle behind the PET scanner.

Leptogenesis, Baryogenesis: asymmetry in the Universe

The Universe began with pure (radiative) energy, and devolved into energy and matter. Matter/antimatter = baryon/anti-baryon $\sim 10^9$ is one mystery.

Here is another mystery: $1/\eta =$ photon/baryon ratio $\sim 10^9$ today as well. Related?

These relations almost certainly a remarkable fossil of the **quark epoch.** When $kT \approx 150$ MeV, $\gamma + \gamma \rightleftharpoons q + \bar{q}$ dominated.

Now anti-quarks make antimatter. Suppose a very tiny asymmetry:

$$\delta_q \equiv \frac{n_q - n_{\bar{q}}}{n_q + n_{\bar{q}}} \ll 1 \qquad (39)$$

As the Universe expands, $q, \bar{q}$ pair production out of the vacuum fades. In every billion quarks produced, say 999,999,997 anti-quarks due to a tiny asymmetry. (We have no real idea what causes this.)

The 3 leftover quarks are surrounded by 2 billion photons. Thus, after the 3 quarks make a baryon, $\eta \sim 1/(2 \times 10^9) \sim 5 \times 10^{-10}$.

Thus, $n_q/n_\gamma \sim \delta_q$. This beautiful, simple argument could explain:

$$n_{\text{anti-baryon}} \ll n_{\text{baryon}} \ll n_{\text{photon}} \qquad (40)$$

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Why BBNS was so inefficient - early neutrons & protons

At t = 0.1 seconds, $T = 3 \times 10^{10}$ K, $\langle E \rangle \approx$ 10 MeV, and neutrons and protons were in equilibrium:

$$n + \nu_e \rightleftharpoons p + e^- \qquad\qquad n + e^+ \rightleftharpoons p + \bar{\nu}_e$$

So both distributions given by the Maxwell-Boltzmann equation

$$n_X = g_X \left(\frac{m_X kT}{2\pi\hbar^2} \right)^{3/2} \exp\left(-\frac{m_X c^2}{kT} \right) \tag{41}$$

where X refers to either n or p, statistical weight $g_X = 2$.

It follows trivially that

$$\frac{n_n}{n_p} = \left(\frac{m_n}{m_p} \right)^{3/2} \exp\left(-\frac{(m_n - m_p)c^2}{kT} \right) = \exp\left(-\frac{Q_n}{kT} \right) \tag{42}$$

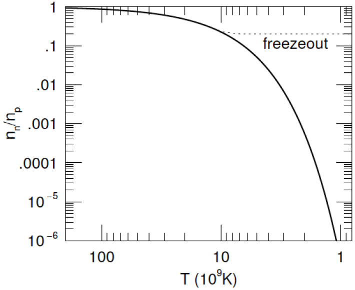
where $Q_n = 1.29$ MeV.

Why BBNS was so inefficient - early neutrons & protons

Detailed calculations (e.g. Ryden's book) show the temperature where neutrons and protons "freeze out" and survive by themselves (i.e. not part of interactions) for the first time is $kT_{\text{freeze}} \approx 0.8$ MeV.

$$\frac{n_n}{n_p} = \exp\left(-\frac{Q_n}{kT_{\text{freeze}}}\right) \approx 0.2 \tag{43}$$

so even as T drops, protons are favoured over neutrons. This lack of neutrons at freeze-out is the primary reason BBNS was so pathetic, i.e. only very light elements formed, mostly H.



BBNS & Y_P = Helium:Hydrogen ratio

Weak force: $p + p \rightleftharpoons D + e^{+} + \nu_e$ (since involves a neutrino)

Weak force: $n + n \rightleftharpoons D + e^{-} + \bar{\nu}_e$ (since involves a neutrino)

Strong force: $p + n \rightleftharpoons D + \gamma$

BBNS is mostly 2-body interactions. Weak force has tiny cross-section, so only relevant in dense places like the Sun's core. $n + n$ is 25 times less likely than $p + p$. **Can you see why?**

Strong force has a much larger cross-section, so higher $p + p$ rate compared to $p + n$ does not help.

Consider random part of Universe at freeze-out when $n_n/n_p \sim 0.2$

To make ^{4}He, then 8 protons left over, i.e. $Y_P = 4/12 = 0.33$ by mass, but this is 50% too high. **Why?**

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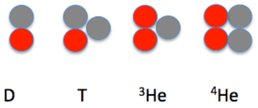
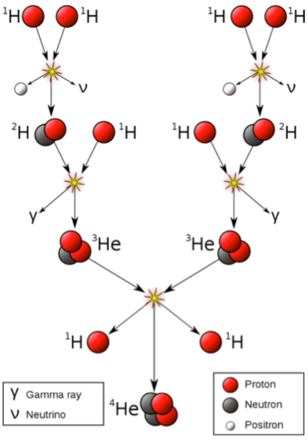
What can you make ?

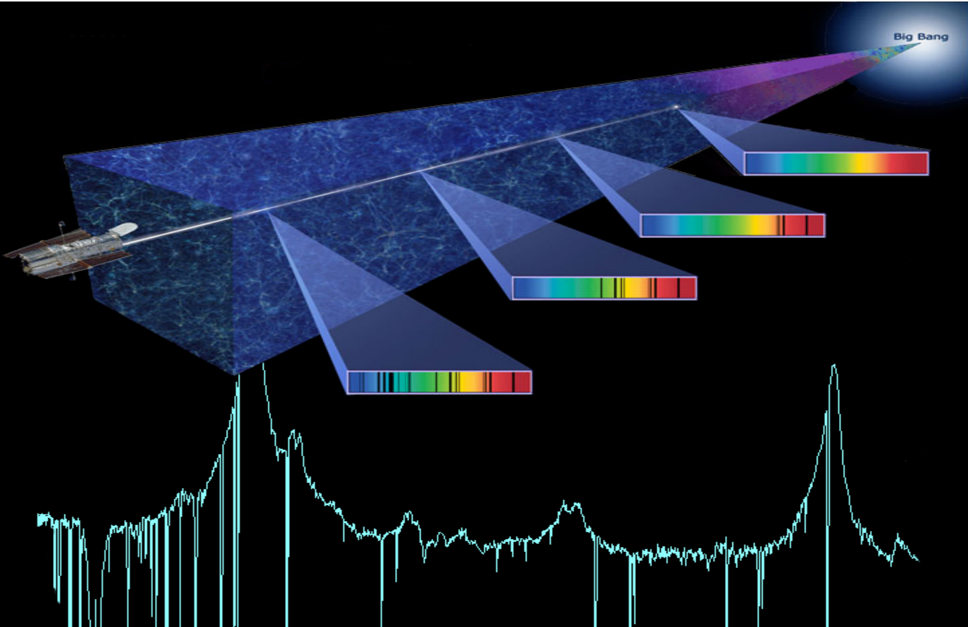


Important fusion reactions & elements



A detailed analysis leads to a primordial value of $Y_P = 0.24$. Today, Y_0 is 15% higher. Why? (Note photons produced; η = photon/baryon ratio is fundamental.)

Measuring Y_P with the Hubble Space Telescope



Towards a better value of Y_P - deuterium

Set the clock at $t = 2$ seconds when $n_n/n_p \approx 0.2$ after freeze-out.
Strong force controls D nucleosynthesis: $p + n \rightleftharpoons D + \gamma$

Energy released, carried off by gamma-ray photon $=$ binding energy (B_D) of D nucleus.
This is a nuclear fusion bomb, i.e. a huge amount of energy per fused particle.

$$B_D = (m_n + m_p - m_D)c^2 = 2.22 \quad \text{MeV} \qquad (44)$$

$$B_D/A = 1.11 \quad \text{MeV} \qquad (45)$$

That photon could also dissociate D, so D cannot have been formed much earlier than $t = 2$ seconds, and presumably gets partly destroyed even at this time. Universal expansion helps.

Following a similar analysis to the above, (n_n, n_p interchangeable)

$$\frac{n_D}{n_p n_n} = 6\left(\frac{m_n kT}{\pi\hbar^2}\right)^{-3/2} \exp\left(\frac{B_D}{kT}\right) \qquad (46)$$

$$\frac{n_D}{n_n} = 6 n_p \left(\frac{m_n kT}{\pi\hbar^2}\right)^{-3/2} \exp\left(\frac{B_D}{kT}\right) \qquad (47)$$

$kT \to \infty$ favours free particles; $kT \to 0$ favours D.

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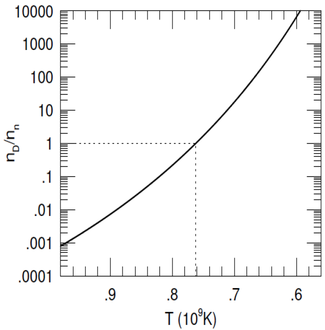
Towards a better value of Y_P - deuterium

Since $n_p \approx 0.8 n_{\text{baryon}}$ and $\eta = n_{\text{baryon}}/n_\gamma$, we can write

$$\frac{n_D}{n_n} = 6.5\eta \left(\frac{kT}{m_n c^2}\right)^{3/2} \exp\left(\frac{B_D}{kT}\right) \tag{48}$$

where we have used $\eta \approx 6 \times 10^{-10}$ (WMAP satellite) and the well known relation for the number of photons arising from a blackbody spectrum (Planck function):

$$n_\gamma = 0.243 \left(\frac{kT}{\hbar c}\right)^3 . \tag{49}$$



If we plot the equation, we can see $n_D/n_n = 1$ at $T \approx 7.6 \times 10^8$ K, or $kT \approx 0.066$ MeV, which happens at $t \approx 200$ seconds. (After this time, too few neutrons to make D.)

From $t = 0.1 \rightarrow 200$ seconds, neutron decay means $n_n/n_p \approx 0.15$ down from 0.2, so $Y_P \approx 0.27$ down from 0.33.

We need to include the other light elements to get to true primordial value, $Y_P = 0.24$. Let's stop here - you get the idea.

Towards a better value of Y_P - deuterium

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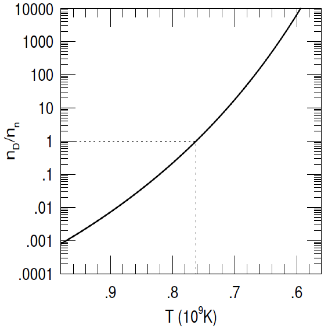

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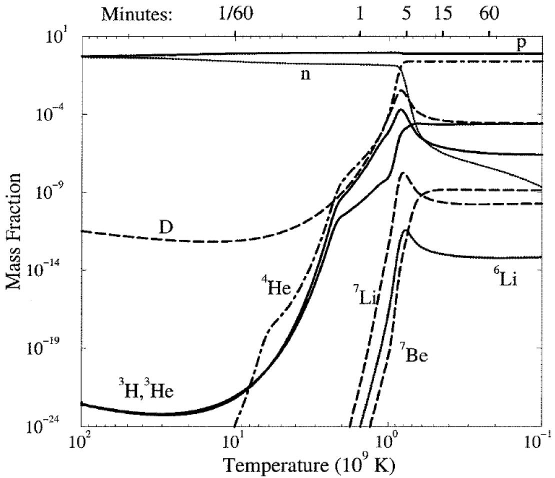
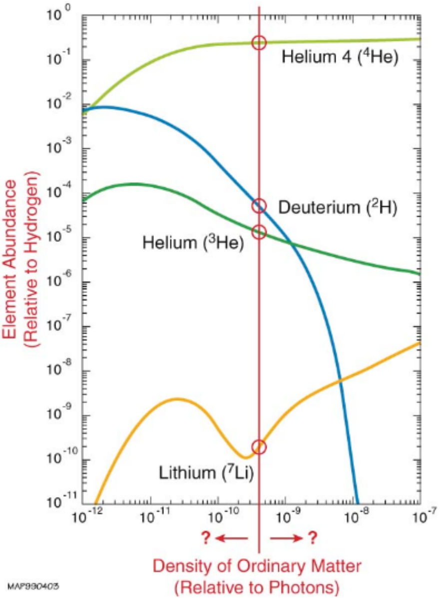
BBNS - all over within a few minutes

Similar arguments can be used to determine T, ^{3}He, ^{6}Li, ^{7}Li, ^{7}Be, (^{5}He, ^{5}Li, ^{8}Be unstable), but basically fusion beyond H gets stuck at tightly bound ^{4}He. (Isotopes in range $5 < A < 8$ mostly unstable.) Getting to $Z = 6$ (Carbon) with its higher B/A is too difficult in a rapidly, adiabatically cooling Universe.



This diagram has a complex dependence on η; ^{4}He goes up, D goes down with increasing η. Of these, D has strongest dependence, so measuring at high redshift so important.

Primordial value of η post quark epoch



You really need to get back to time before stars to be sure D is measuring η. Reason? Stars convert D through their own nucleosynthesis. We need the highest redshift quasars.

Entirely different approach: measure ^{7}Li in the most ancient stars ($\tau \gtrsim 13$ Gyr) in the Galactic halo – near field cosmology!

(Bland-Hawthorn & Freeman 2000; Freeman & Bland-Hawthorn 2002; Bland-Hawthorn & Peebles 2006; Karlsson, Bromm & Bland-Hawthorn 2013; Bland-Hawthorn & Freeman 2014; Bland-Hawthorn & Gerhard 2016)

Universal plasma before Universal fog

Most of the H and He were free-floating protons, He nuclei, electrons, **not** neutral gas atoms – Universe was in an ionized state since too hot (particles moving too quickly, photons too energetic) for a few hundred thousand years. We can work out when the Universal gas turned neutral. And this was all long before the first astrophysical objects.