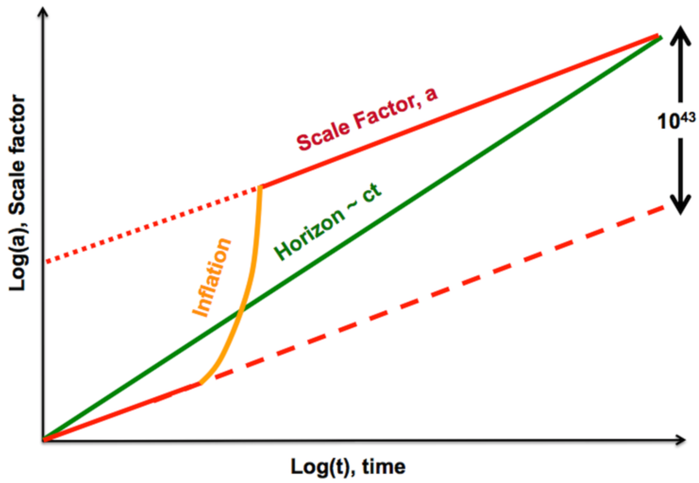
PHYS 2013/2913/2923 Tutorial 3

First, a nice question on the last Assignment. Congrats! So the Big Bang fills the tiny Universe with some kind of radiation and it expands at a slow rate for a split second. Then inflation happens - this is an emergent property from the vacuum (it cannot have zero energy due to Heisenberg's Uncertainty Principle) where the Universe takes off at a HUGE expansion rate and undergoes a hundred e-folds or so.

But why does inflation end and not keep going?? Great question.



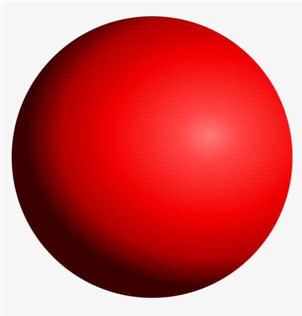
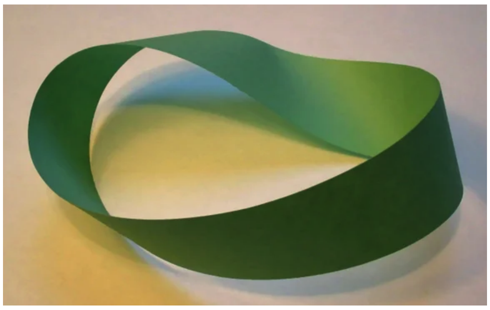
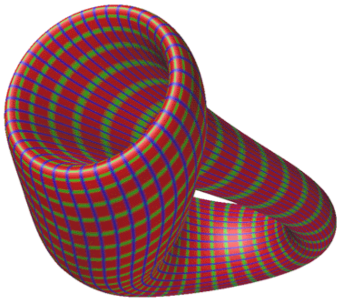
Did you notice an omission in my inflation story so far? Few people noticed this after Alan Guth, but Andrei Linde did. The HUGE inflated Universe must have been EXTREMELY cold. Any particles before inflation (like weird monopoles) must have become extremely rare.

So how do we end up with a Universe today filled with matter and energy? Another emergent property of the vacuum was **reheating**. The quantum vacuum fluctuations that kept appearing on Planck scales ($\ell_P = \sqrt{\hbar G/c^3} = 1.62 \times 10^{-35}$ m) every time the Universe got stretched gave rise to zillions of particles. This process killed off inflation (Universe now filling up with matter and energy) and took us back to the former energy state where we continue to expand at the slow former rate, the original Hot Big Bang. (This is an advanced topic about scalar fields that we discuss in PHYS4122.)

TOPOLOGY, CURVATURE, TIME & SPACE

Here we look at some issues that students struggle with. All students (and scientists) struggle with the strange world of curvature and topology in general. Thus, the impact on things like cosmological distances, time passed, redshift, luminosity, etc. all become a little tricky but you can certainly understand the basic ideas easily enough.

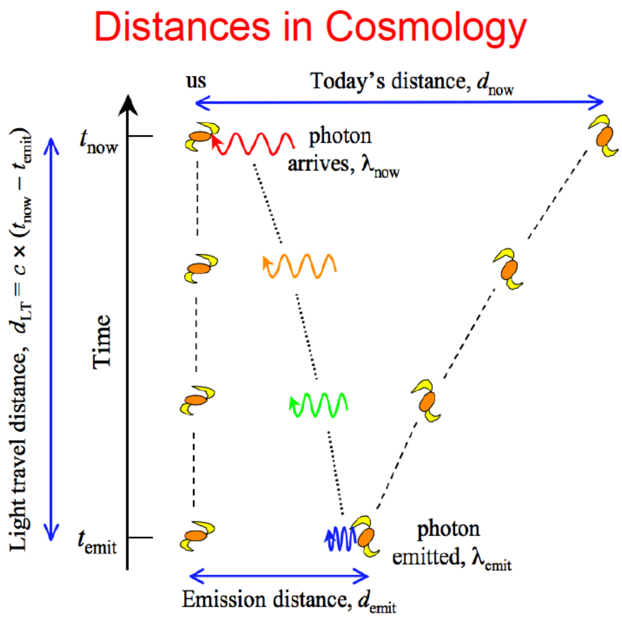
Let me give one example. You can manufacture a path that goes forever without end.

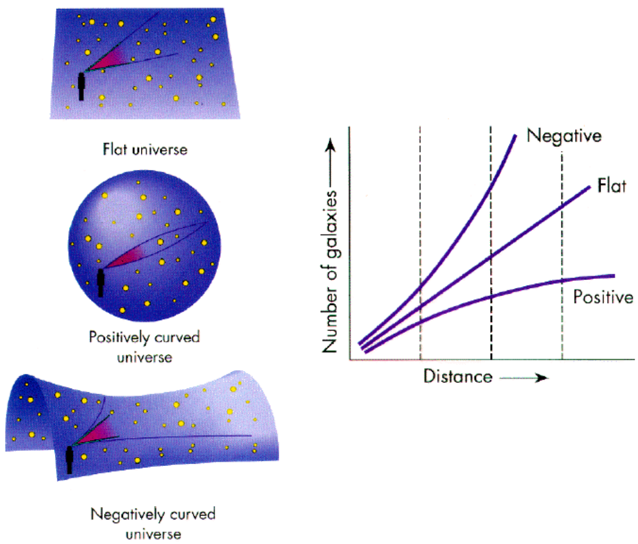
So why would you think of asking what is the Universe expanding into? What appears infinite in a few dimensions can be finite in higher dimensions, especially in a $\kappa > 0$ Universe. (We normally speak of $\kappa = 0$ and $\kappa < 0$ space as being infinite.) There are still people who think the world is flat because they can't see how people can be standing upright on a sphere. They don't buy that the local perception of gravity defines your up and down.

So don't worry about the overall shape of the Universe - it's huge and beyond our horizon for now. (There are far more complex topologies - look up Calabi-Yau or Kaluza-Klein, both crazy but with beautiful properties.)

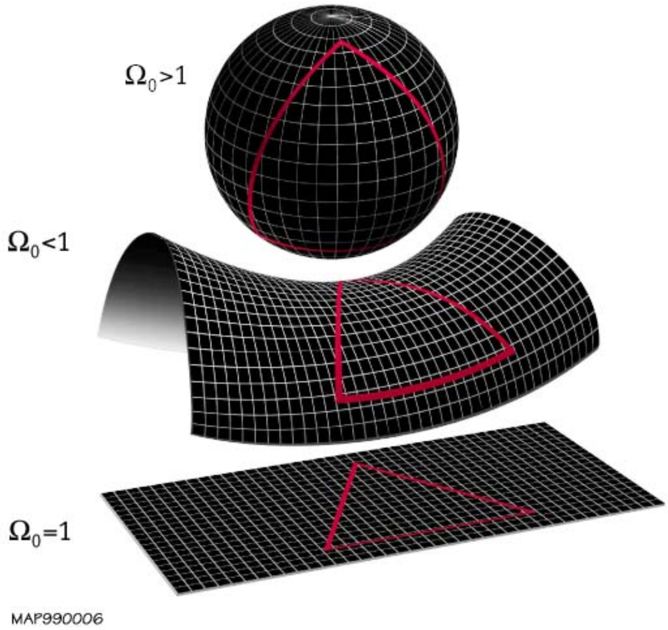
Curvature on the very largest scales is a real thing that modern day astronomers and cosmologists are probing easily. It affects distances, galaxy properties and galaxy counts.



Galaxy counts increase with distance in a given solid angle:



Consider a triangle on different curved surfaces:

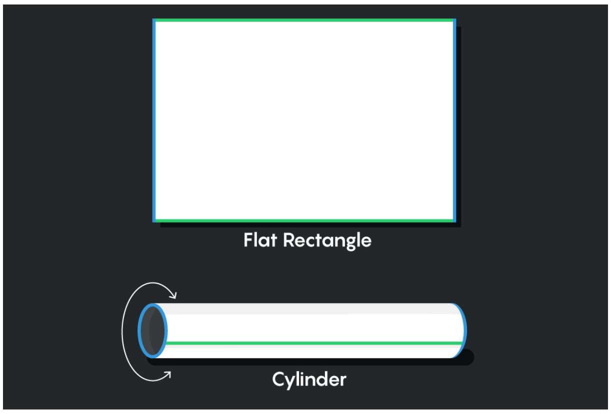


Here are the angle formulae:

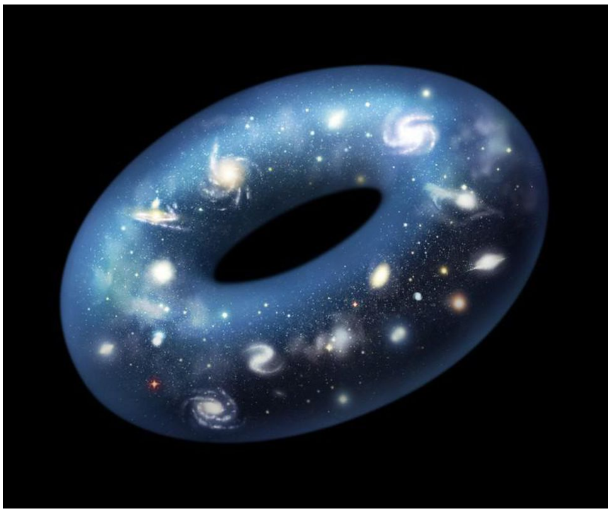
$$\alpha + \beta + \gamma = \pi + A/R^2 \quad (\kappa > 0) \tag{1}$$
$$\alpha + \beta + \gamma = \pi \quad (\kappa = 0) \tag{2}$$
$$\alpha + \beta + \gamma = \pi - A/R^2 \quad (\kappa < 0) \tag{3}$$

where A is the area of the triangle and R is the radius of curvature. (A/R^2 is a solid angle since the total surface gives you 4π. This is why there is a $d\Omega$ term in the Robertson-Walter metric, i.e. you are correcting angles and distances for curvature.)

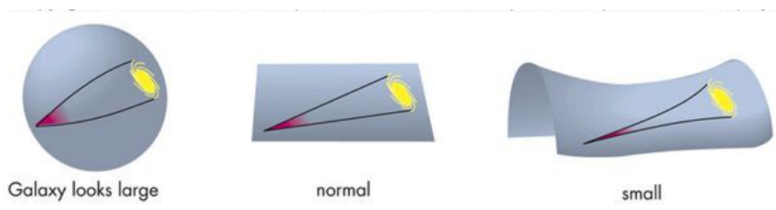
If the angles add up to π, does that mean we live in a flat Universe?



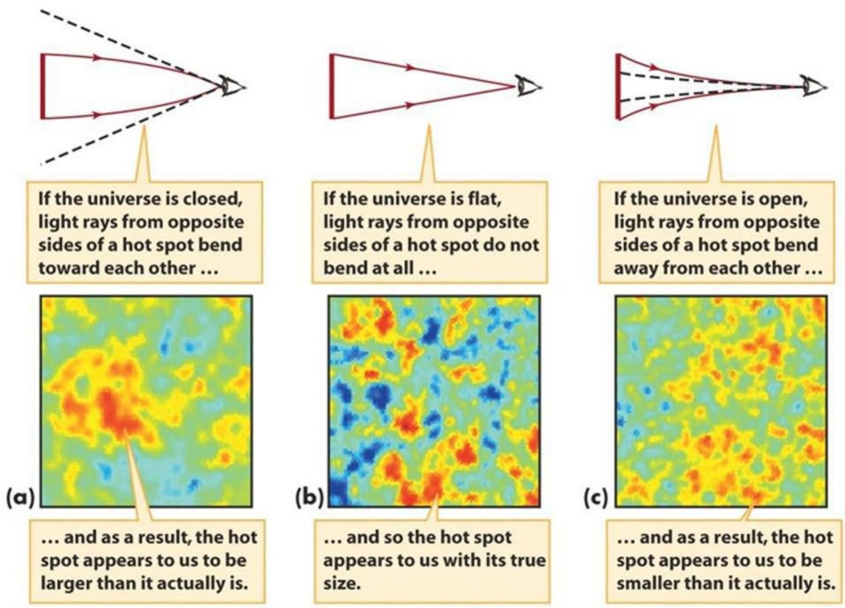
This extraordinary object has $\kappa > 0$ on outer side, $\kappa < 0$ on inner surface, so angles can be close to π.



What about perceived angles with distance?

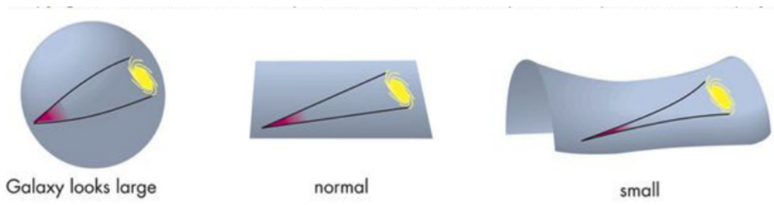


And this affects our interpretation of the blobs in the CMB:



Can you now see how that affects perceived luminosity with distance? (flux through a solid angle)

What about perceived angles with distance?



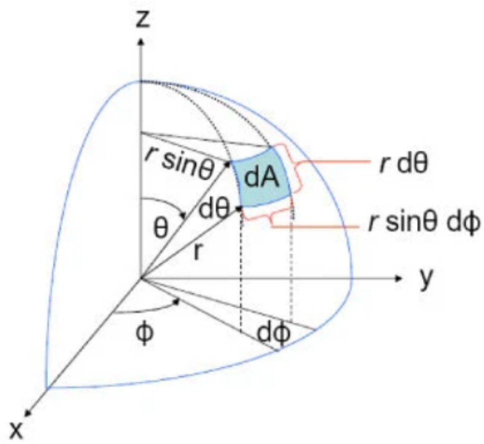
On a 2D flat plane, going from (x, y) to $(x + dx, y + dy)$, we can write the line segment is $d\ell^2 = dx^2 + dy^2$. But we normally work in polar or spherical coordinates. Going from (r, θ) to $(r + dr, \theta + d\theta)$, the line segment is $d\ell^2 = dr^2 + r^2 d\theta^2$. This is for $\kappa = 0$.

In a 2D curved space (R is radius of curvature), the line segments are:

$$d\ell^2 = dr^2 + R^2 \sin^2(r/R) d\theta^2 \quad (\kappa > 0) \tag{4}$$

$$d\ell^2 = dr^2 + R^2 \sinh^2(r/R) d\theta^2 \quad (\kappa < 0) \tag{5}$$

For $r \ll R$, all formulae are identical of course.



In 3D flat space, going from (x, y, z) to $(x + dx, y + dy, z + dz)$, we can write the line segment is $d\ell^2 = dx^2 + dy^2 + dz^2$. Going from (r, θ, ϕ) to $(r + dr, \theta + d\theta, \phi + d\phi)$, the line segment is $d\ell^2 = dr^2 + r^2(d\theta^2 + \sin^2\theta \, d\phi^2)$. This is for $\kappa = 0$.

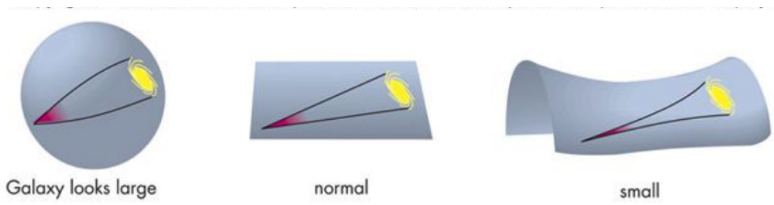
In 3D curved space, the line segments are:

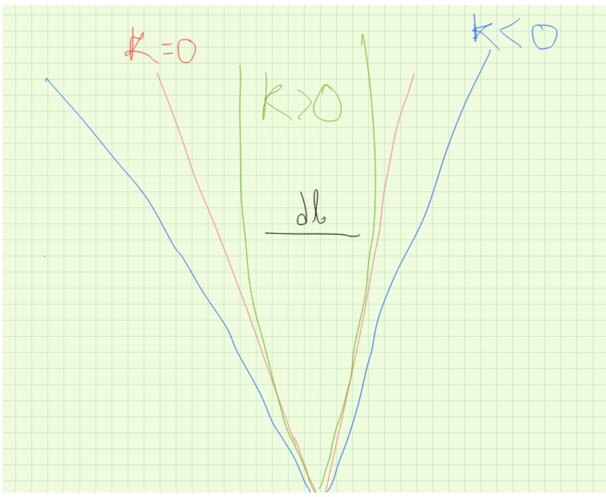
$$d\ell^2 = dr^2 + R^2 \sin^2(r/R)(d\theta^2 + \sin^2\theta \, d\phi^2) \quad (\kappa > 0) \tag{6}$$

$$d\ell^2 = dr^2 + R^2 \sinh^2(r/R)(d\theta^2 + \sin^2\theta \, d\phi^2) \quad (\kappa < 0) \tag{7}$$

For $r \ll R$, all formulae are identical of course.

What about perceived angles with distance? This gets us to the **angular diameter distance.**





So we need formulae for the 2d space (the best way to think about all this stuff):

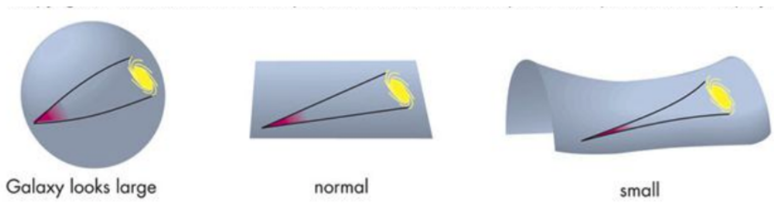
$$d\ell^2 = dr^2 + R^2 \sin^2(r/R) d\theta^2 \quad (\kappa > 0) \tag{8}$$
$$d\ell^2 = dr^2 + r^2 d\theta^2 \quad (\kappa = 0) \tag{9}$$
$$d\ell^2 = dr^2 + R^2 \sinh^2(r/R) d\theta^2 \quad (\kappa < 0) \tag{10}$$

For $r \ll R$, all formulae are identical of course. If we simply want to consider angle, i.e. (r, θ) to $(r, \theta + d\theta)$,

$$d\ell = R \sin(r/R) d\theta \quad (\kappa > 0) \tag{11}$$
$$d\ell = r\, d\theta \quad (\kappa = 0) \tag{12}$$
$$d\ell = R \sinh(r/R) d\theta \quad (\kappa < 0) \tag{13}$$

We must now include Universal expansion because this is a separate effect from the effects of distance. If the Universe is static, $d\ell \propto r\, d\theta$; if not, we must include the scale factor a.

And now for the **angular diameter distance**, d_A:



We can reduce all three formulae to:

$$\begin{aligned} \ell &= a(t_e)\, S_\kappa(r)\, \delta\theta && (14) \\ &= \frac{S_\kappa(r)\, \delta\theta}{1+z} && (15) \end{aligned}$$

for a standard yardstick ℓ, so that we write

$$d_A \equiv \frac{\ell}{\delta\theta} = \frac{S_\kappa(r)}{1+z} \tag{16}$$

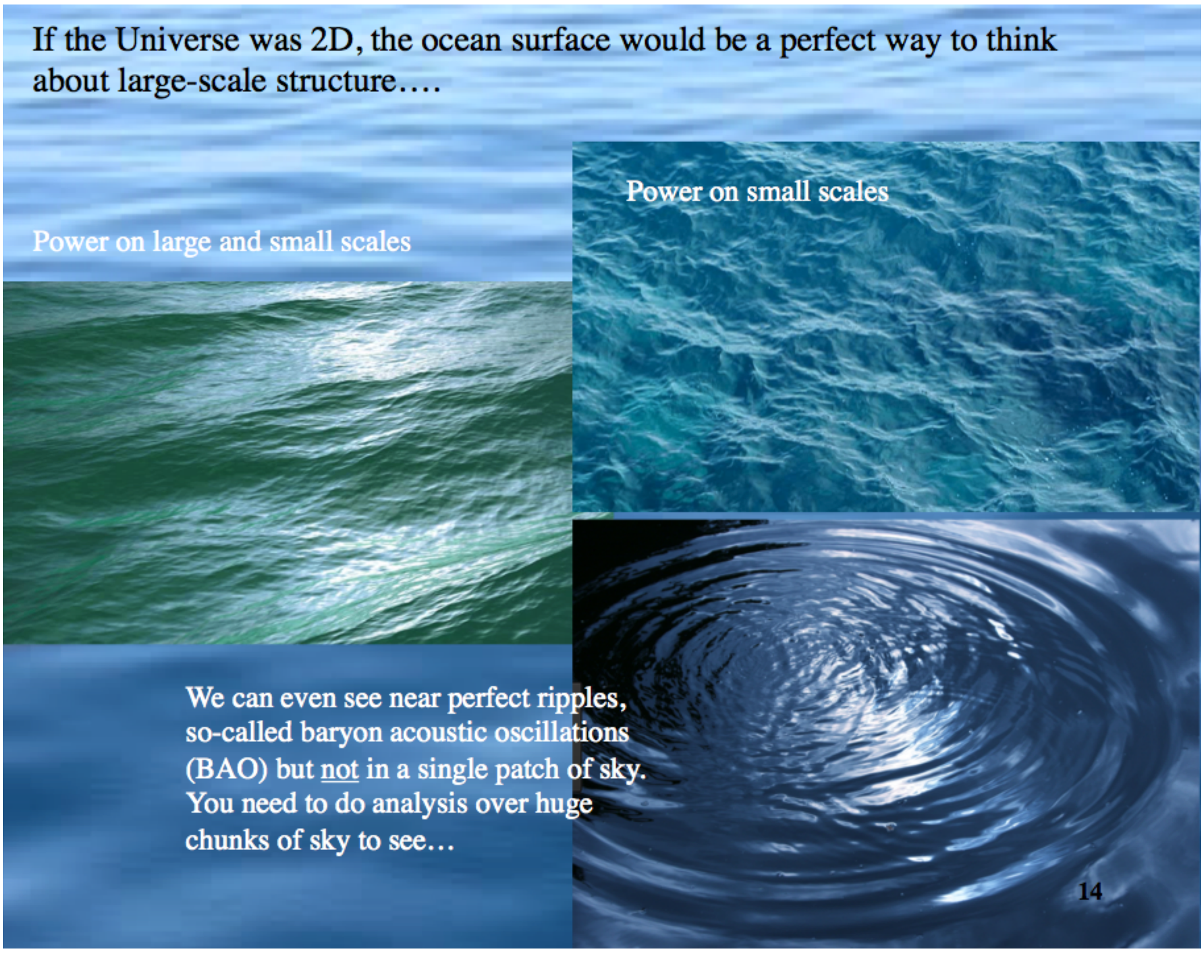
This next step is subtle, but important. Cosmic time is **proper time**. In a flat Universe, the angular-diameter distance is **not** equal to **proper distance** today to the source, but **proper distance** at the time the light was emitted.

$$d_A \equiv d_P(t_e) = \frac{d_P(t_0)}{1+z} \tag{17}$$

This is obvious enough - imagine that two fireflies flashed at you when they were 1 degree apart as seen against the night sky. That angle is preserved for all time since the message was sent, even though the fireflies are flying away from you after that and seem to be converging.

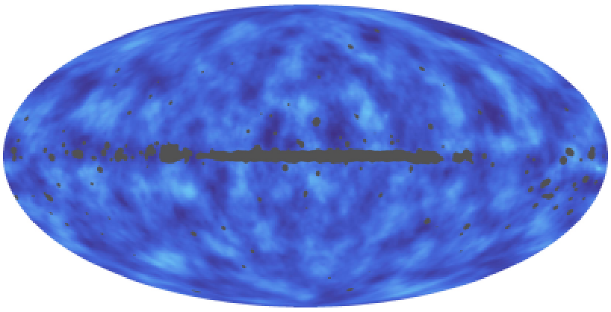
So when the entire Universe was small, the CMB flashed the surface of last scattering at us; it really was a narrow interval in cosmic time. Today, that scattering surface is 1000$\times$ further away, but the angular scale is preserved for all time.

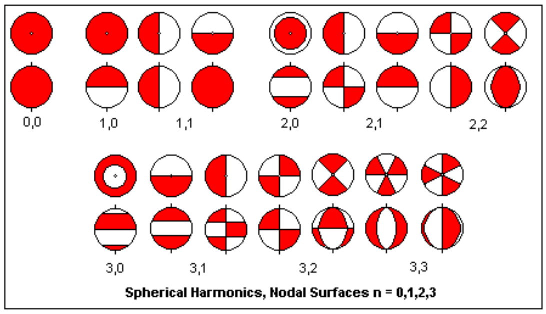
Revisit how we interpret the cosmic microwave background.


If the Universe was 2D, the ocean surface would be a perfect way to think about large-scale structure….
Power on small scales
Power on large and small scales
We can even see near perfect ripples, so-called baryon acoustic oscillations (BAO) but not in a single patch of sky. You need to do analysis over huge chunks of sky to see…
14

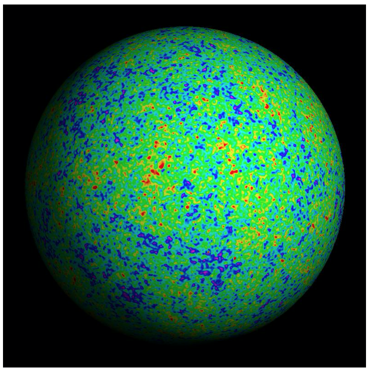
Estimating the power spectrum of the CMB

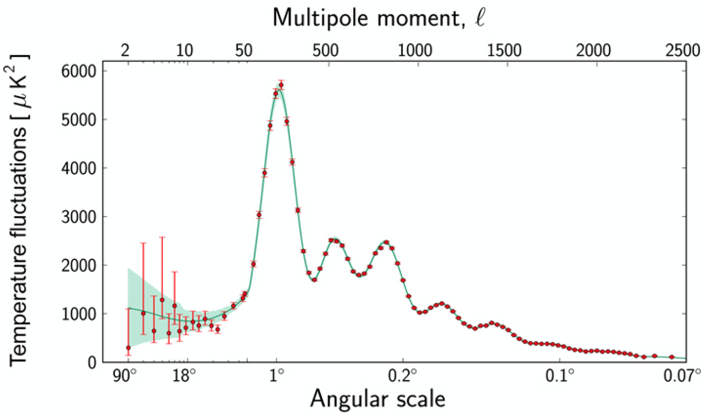
The LHS image is boosted to show you structure on one angular scale. You basically multiply by the weights on RHS to find the power spectrum using web tools freely available. Power at 18° is much less than the power at, say, 1°.





The peaks relate to cosmological parameters - see next slide.

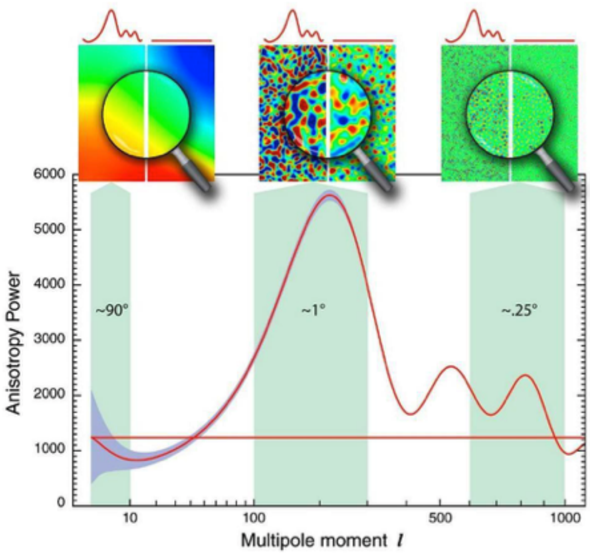


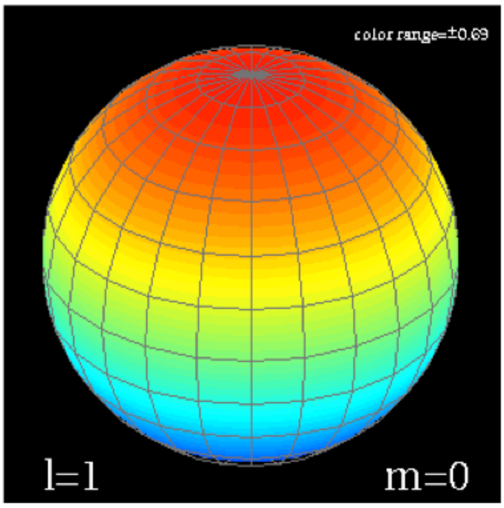


THE UNIVERSITY OF SYDNEY

Spherical harmonics over a 2D surface

The power spectrum for all angles on the sky is shown below. The magnifying glass shows signal vs. no signal.





There are very nice web tools that show how to fit this spectrum, and relate to cosmological values.

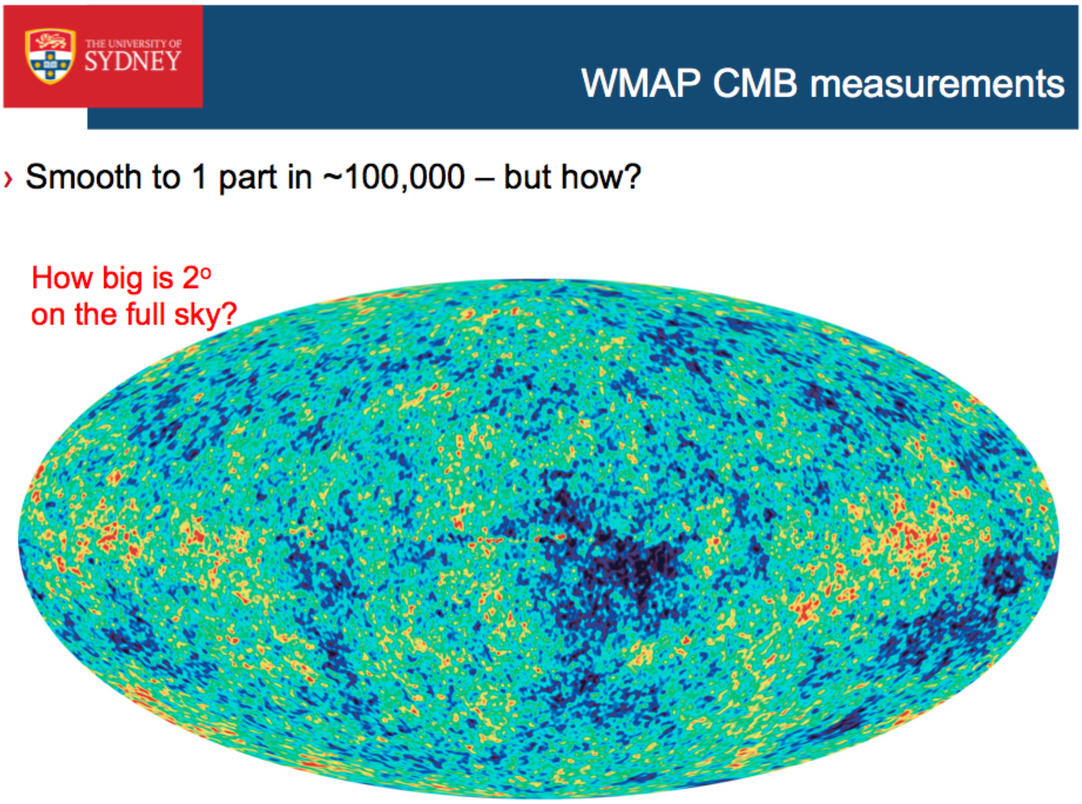
Answers on the next slide.

THE UNIVERSITY OF SYDNEY

WMAP Nine Year Results – reading off the sky !!

Best-fit cosmological parameters from WMAP nine-year results[16]

Parameter	Symbol	Best fit (WMAP only)	Best fit (WMAP + eCMB + BAO + H_0)
Age of the universe (Ga)	t_0	13.74 ±0.11	13.772 ±0.059
Hubble's constant ($^{km}/_{Mpc \cdot s}$)	H_0	70.0 ±2.2	69.32 ±0.80
Baryon density	Ω_b	0.0463 ±0.0024	0.046 28 ±0.000 93
Physical baryon density	$\Omega_b h^2$	0.022 64 ±0.000 50	0.022 23 ±0.000 33
Cold dark matter density	Ω_c	0.233 ±0.023	$0.2402^{+0.0088}_{-0.0087}$
Physical cold dark matter density	$\Omega_c h^2$	0.1138 ±0.0045	0.1153 ±0.0019
Dark energy density	Ω_Λ	0.721 ±0.025	$0.7135^{+0.0095}_{-0.0096}$
Density fluctuations at 8h^{-1} Mpc	σ_8	0.821 ±0.023	$0.820^{+0.013}_{-0.014}$
Scalar spectral index	n_s	0.972 ±0.013	0.9608 ±0.0080
Reionization optical depth	τ	0.089 ±0.014	0.081 ±0.012
Curvature	$1-\Omega_{\text{tot}}$	$-0.037^{+0.044}_{-0.042}$	$-0.0027^{+0.0039}_{-0.0038}$
Tensor-to-scalar ratio (k_0 = 0.002 Mpc^{-1})	r	< 0.38 (95% CL)	< 0.13 (95% CL)
Running scalar spectral index	$dn_s/d\ln k$	−0.019 ±0.025	−0.023 ±0.011



So this is where Alan Guth's cosmic inflation (1981) came to the rescue. The

Universe was once tiny, in full causal contact, then went through a phase transition when the forces emerged, inflating enormously in a tiny fraction of a second (recall your 2nd Assignment).

Why inflation?

Guth's proposal for cosmic inflation solved a number of fundamental problems in one go: magnetic monopole problem, horizon problem, flatness problem, etc.

I will address one of these here, **the horizon problem.** When we look at the CMB, how far was the observable horizon at $z = 1100$ (Hubble distance)? Recombination was during the matter-dominated era, such that

$$(\frac{H(z)}{H_0})^2 = \Omega_{M,0}(1+z)^3 \tag{18}$$

where H_0 and $\Omega_{M,0}$ are defined today, so

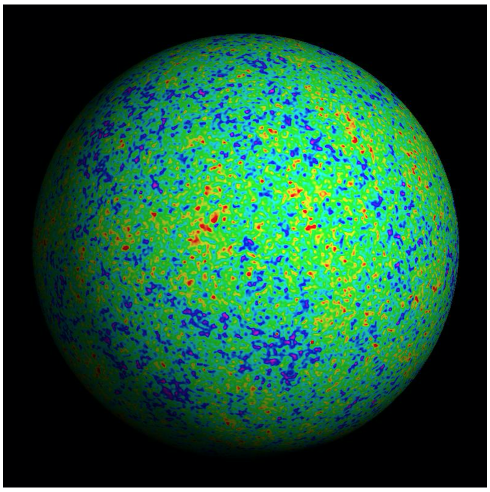
$$\frac{c}{H(z_{\rm CMB})} = \frac{3 \times 10^8 \text{ m s}^{-1}}{1.24 \times 10^{-18} \text{ s}^{-1}(1101)^{3/2}} = 0.2 \text{ Mpc} \tag{19}$$

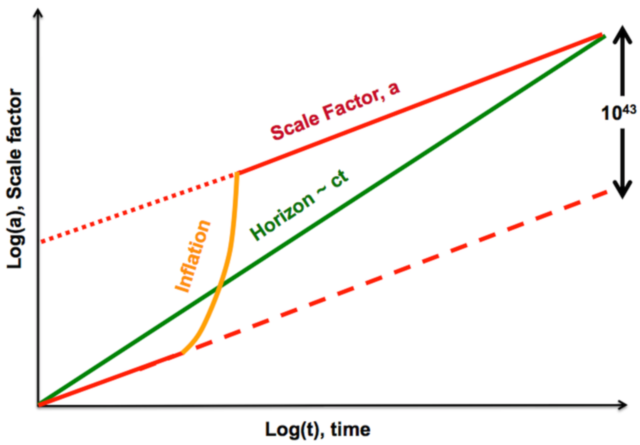
To get the angle on the sky, we divide by the angular diameter distance, d_A, such that

$$\theta_H = \frac{c/H(z_{\rm CMB})}{d_A} \approx \frac{0.2 \text{ Mpc}}{13 \text{ Mpc}} \approx 1^\circ \tag{20}$$

That's about the size of one of the blobs in the CMB (RHS). Today, that maps to something bigger than the Virgo cluster, say.

So why is there such incredible uniformity across the entire Universe at $z = 1100$ **given that none of it has been able to communicate?** The Universe was once tiny, in full causal contact, then inflated hugely onto a scale that could never be in causal contact (faster than light).





The particle horizon

› Q: If the Universe has age t, what is the furthest distance we can see?

radial path of a photon: $$\int_0^r \frac{dr}{[1-kr^2]^{1/2}} = \int_0^t \frac{cdt}{a(t)}$$

› Look at the solution for EdS case:

$$a(t) = a_0 \left(\frac{t}{t_0} \right)^{2/3}$$

Matter dominated

$$\mathrm{d}_\mathrm{H}(t) = ra(t) = a(t) \int_0^t \frac{cdt'}{a(t')} = a_0 \left(\frac{t}{t_0} \right)^{2/3} \int_0^t \frac{cdt}{a_0 (t/t_0)^{2/3}}$$

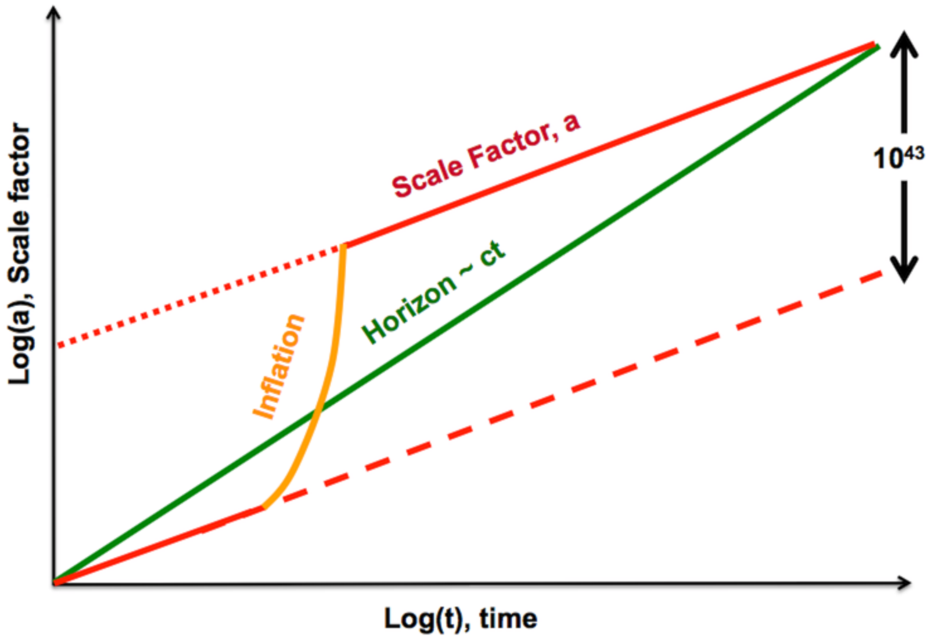
The particle horizon

$$d_H(t) = ct^{2/3}\int_0^t \frac{dt}{t^{2/3}} = ct^{2/3}\left[3t^{1/3}\right] = \boxed{d_H(t) = 3ct}$$

› This is the furthest ***proper*** (=comoving at t_0) distance that an event can currently be observed and is called the **Particle Horizon**.

› In this case the scale factor, a(t), increases at $t^{2/3}$, but the horizon increases faster.

› The fraction of the Universe that is visible, ***and in causal contact***, increases with time, and decreases towards the Big Bang.



The CMB and horizons

- Q: Why does the CMB look so uniform?
- The particle horizon gives the maximum distance that a photon can propagate?

$$d_H(t) = 3ct \quad \text{for EdS Universe}$$

$$\text{then use} \quad a(t) = a_0\left(\frac{t}{t_0}\right)^{2/3} \qquad \text{so } d_H(a) = 3ct_0\left(\frac{a}{a_0}\right)^{3/2}$$

$$\text{and recall } t_0 = \frac{2}{3H_0} \qquad \text{so} \quad \boxed{d_H(z) = \frac{2c}{H_0}\frac{1}{(1+z)^{3/2}}}$$

This horizon distance (proper distance) at z=1100 was not very large, in fact less than ~ 200 kpc. You need to convert H_o = 70 km/s/Mpc to SI units to see that trivially, and recall that 1 parsec = 3 x 10^{16} m.

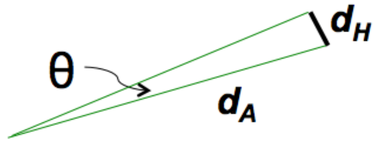
That's a smaller sphere than the outer halo of the Milky Way today.

THE UNIVERSITY OF SYDNEY

The CMB and horizons

- Now we have the particle horizon at redshift z, we can also calculate the angular size of the horizon (recalling the angular diameter distance, d_A):

$$\theta = \frac{d_H(z)}{d_A(z)} = \frac{\text{(proper horizon distance at z)}}{\text{(angular diameter distance to z)}}$$



For an EdS Universe:

$$d_A(z) = \frac{d_c(z)}{(1+z)} = \frac{2c}{H_0}\left[1 - \frac{1}{(1+z)^{1/2}}\right]\frac{1}{(1+z)} \quad \text{and} \quad d_H(z) = \frac{2c}{H_0}\frac{1}{(1+z)^{3/2}}$$

$$\text{So } \theta = \frac{1}{(1+z)^{1/2}}\left[\frac{1}{1 - 1/(1+z)^{1/2}}\right]$$

CMB decoupling is at z=1100, so

θ= 0.03 radians = 1.7 degrees

- Areas of the CMB separated by more than ~1.7 deg have not been in causal contact. ***Why do they look the same?***

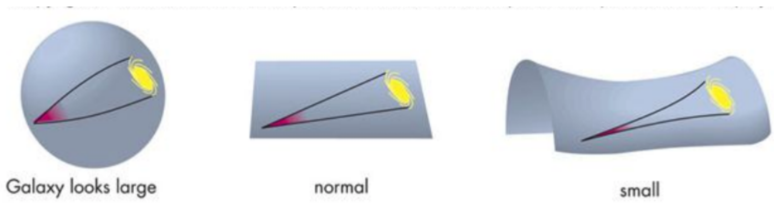
So the angular diameter distance to the surface of last scattering today is about 13 Mpc.

The proper distance is more like 13,000 Mpc, i.e. almost the size of the observable Universe.

Luminosity distance is, in principle, easier to understand. This time we use a standard candle (like a 100W lightbulb or a Type Ia supernova) and write the simple relation:

$$d_L \equiv \left(\frac{L}{4\pi f}\right)^{1/2} \tag{21}$$

Remember what happens to solid angles:



So there's a $1/(1+z)$ correction for the loss of energy due to z, a $1/(1+z)$ correction for time being stretched (energy per unit time) by z, thus

$$f = \frac{L}{4\pi S_\kappa(r)^2(1+z)^2} \tag{22}$$

where the correction for curvature is now included as well. So thus

$$\begin{aligned} d_L &= S_\kappa(r)(1+z) && (23) \\ &= r(1+z) \;\; (\kappa = 0, \text{our Universe}) && (24) \\ &= d_P(t_0)(1+z) && (25) \end{aligned}$$

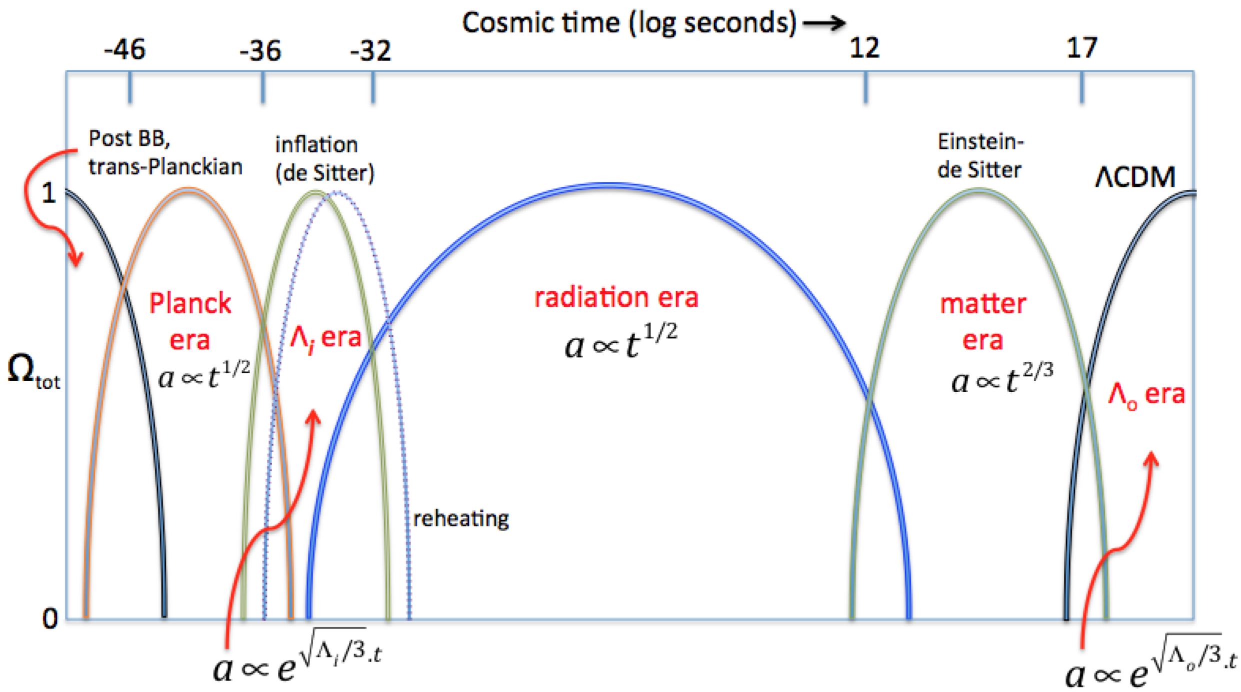
Note: if you estimate the distance to a standard candle using the inverse square law, you will overestimate the proper distance by a factor $(1+z)$.

Useful connections ($\kappa = 0$):

$$d_A(1+z) = d_P(t_e)(1+z) = d_P(t_0) = \frac{d_L}{1+z} \tag{26}$$

Overview & revision

The Universe is a remarkable study that begins with physics we don't understand (trans-Planckian), moving through QFT (inflation), GR, particle physics, leptogenesis, baryogenesis, and astrophysics once matter takes over.



Remember that the Hubble parameter $H = \frac{\dot{a}}{a}$ so this has a different dependence in each era, declining overall like everything else, e.g. energy/matter (non-Λ) density declining like a^{-3}, temperature, pressure, etc. Note that during the Λ eras, H is a constant. Expanding universes have changing a but not necessarily H.