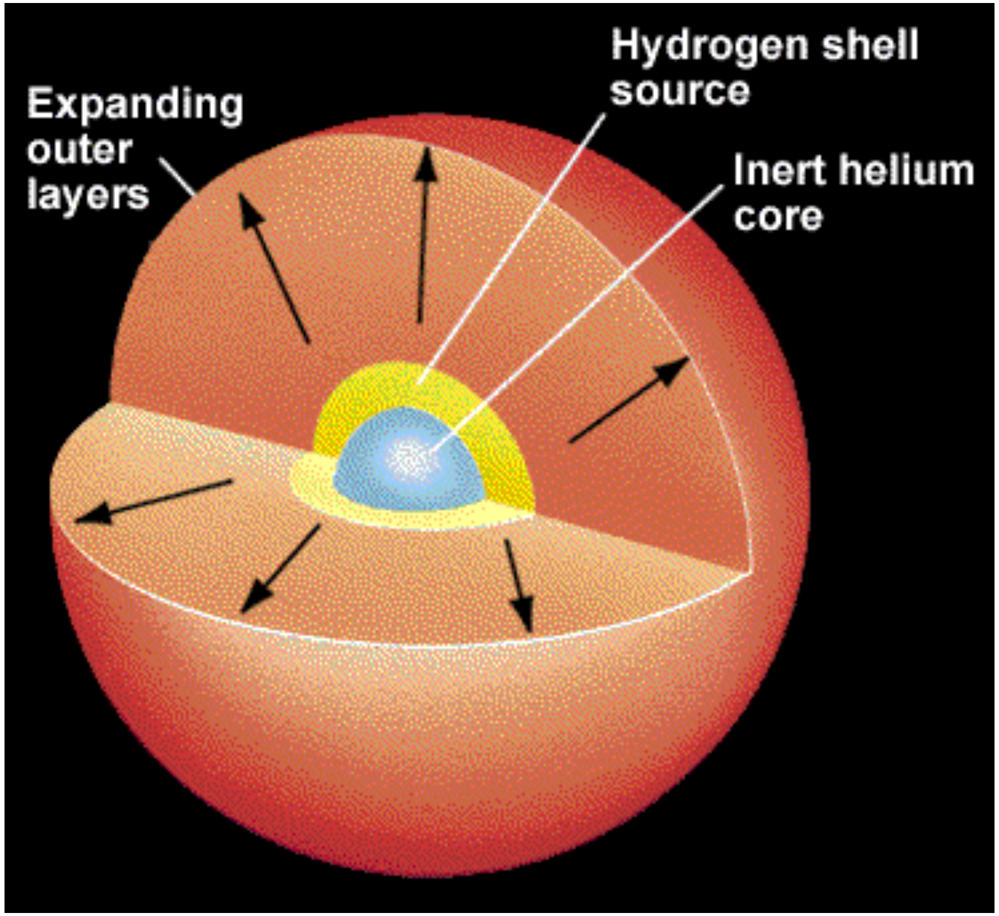
PHYS 2013/2913/2923
Astrophysics & Cosmology

Tutorial 4 – Joss Bland-Hawthorn

Most of this is revision, in response to nice questions emailed to me. I will focus on easy ways to derive the cosmological formulae, e.g. distances, ages.

Jeans instability, mass, length

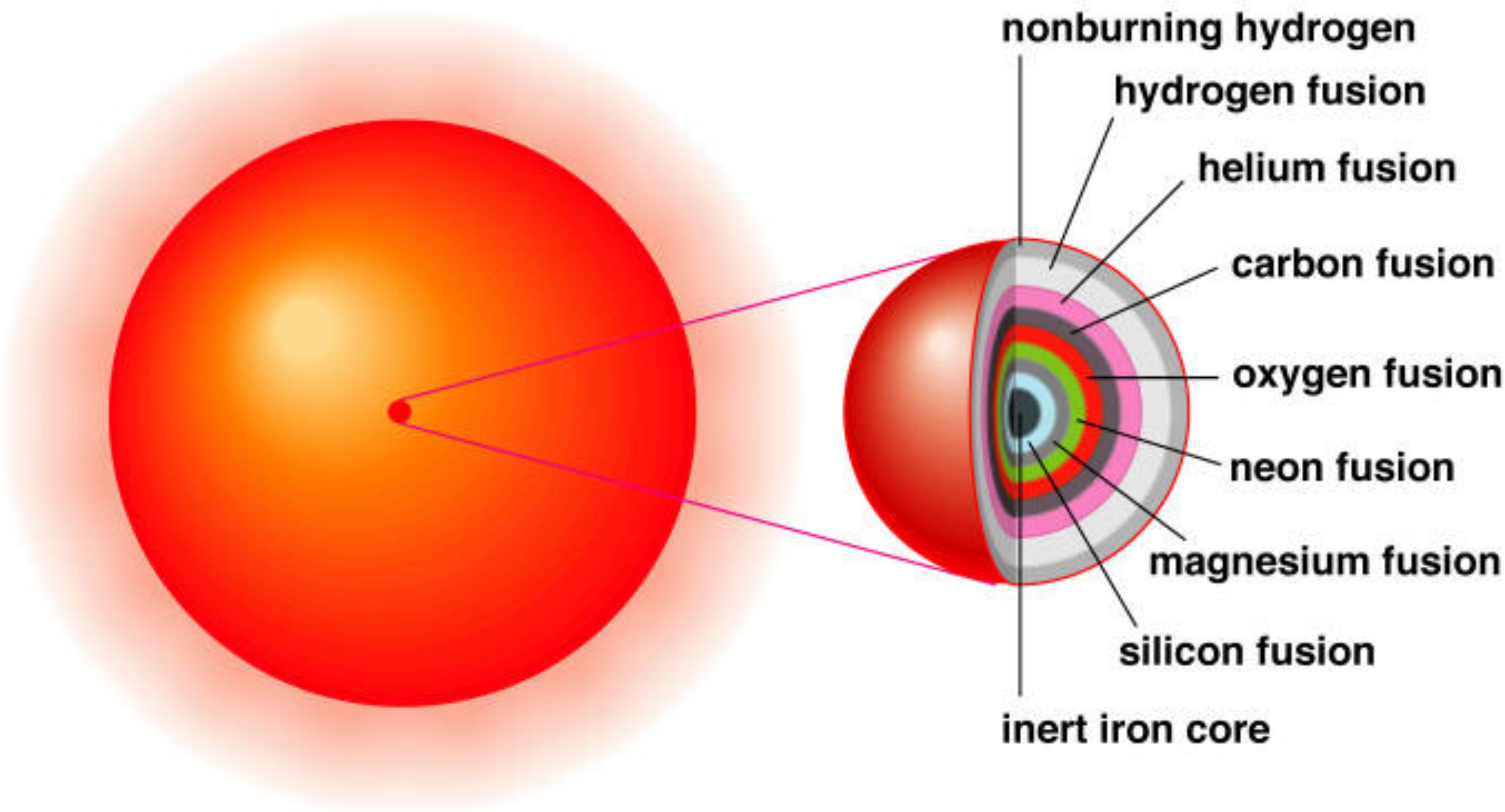
Heavy elements in stars, fallback



Nuclear fusion leads to onion rings in evolving stars…

It stops at Fe because the binding energy per nucleon is at its peak.

The star collapses since nothing holding it up, big bounce, supernova !



You can see now that when the bounce and explosion happens, the innermost shells are the ones that fall back (heavier elements) – evidence of black holes in first stars.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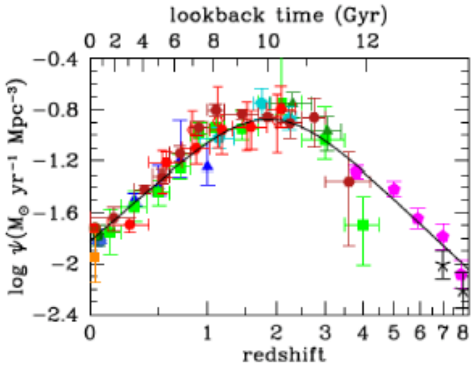
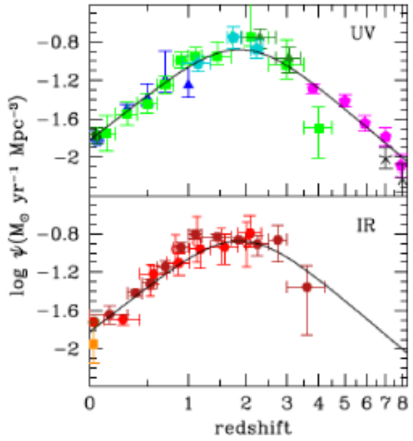
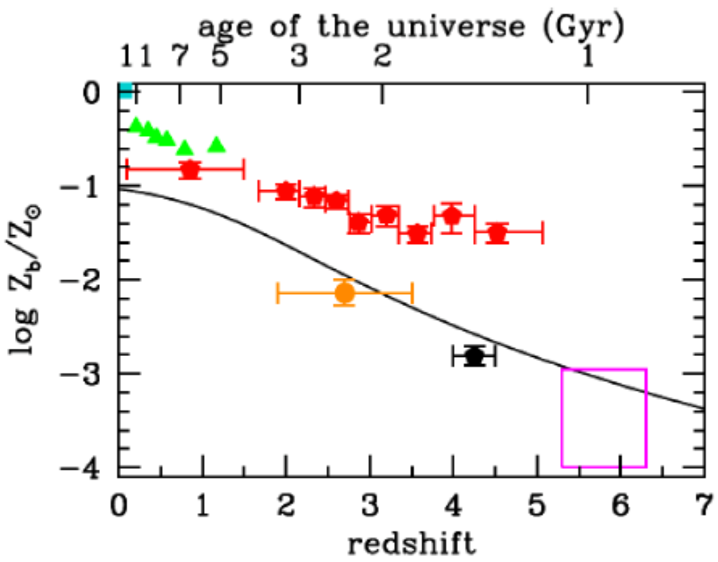
The Golden Age, $5 < z < 1$ $(1 < t_{\mathrm{age}}/\mathrm{Gyr} < 5)$

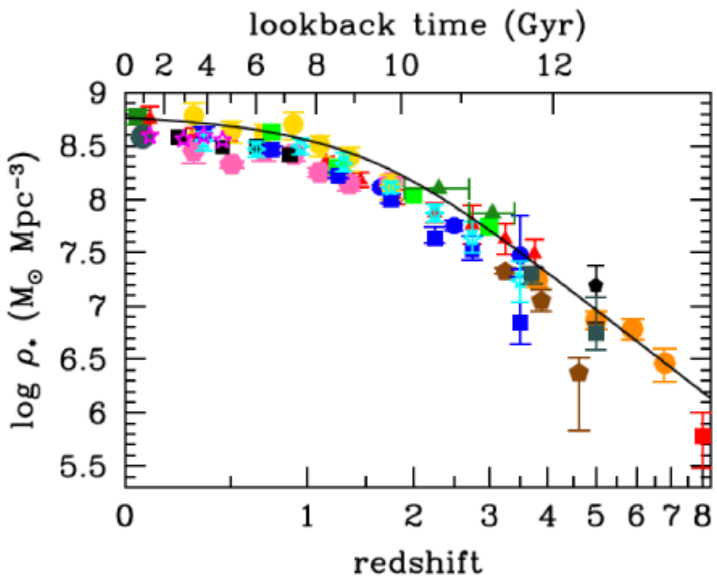
Madau & Dickinson (2014) review what we know about the star formation history of the Universe. **This is the key to understanding galaxy evolution.** The star formation rate ψ (SFR) per comoving Mpc^3 per year went through a peak 10 Gyr ago. We see the same trend at all wavelengths.

Consistent with that, an increasing fraction of baryons $\rho_\star$ is locked up in stellar mass (bottom RHS), and the metal fraction $Z/Z_\odot$ is increasing also to $z = 0$ (bottom LHS).









The Golden Age, $5 < z < 1$ $(1 < t_{\rm age}/{\rm Gyr} < 5)$

There was a lot more gas around in a much smaller Universe. Everything was happening then. See how black hole (accretion disc) activity also peaked then (bottom LHS). The star formation efficiency per unit mass ψ/M (a meaure of SF efficiency) has been in decline ever since (bottom RHS), mostly because gas accretion has been drying up.

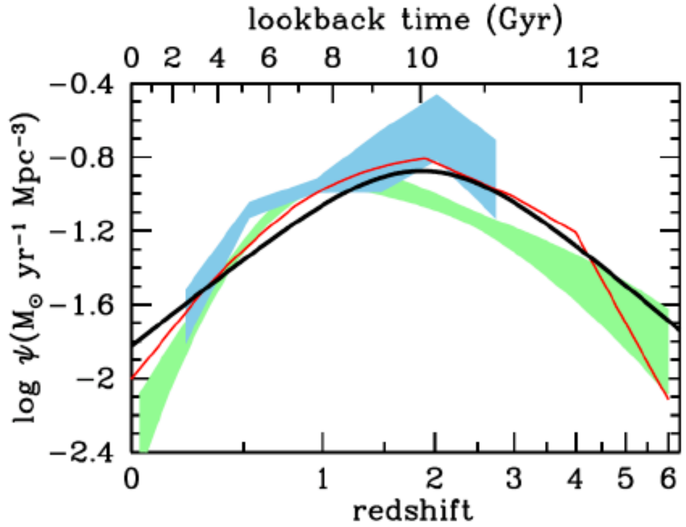


Figure 15: Comparison of the best-fit star formation history (*thick solid curve*) with the massive black hole accretion history from X-ray (Shankar et al. 2009, *red curve*; Aird et al. 2010, *light green shading*) and infrared (Delvecchio et al. 2014, *light blue shading*) data. The shading indicates the $\pm 1\sigma$ uncertainty range on the total bolometric luminosity density. The radiative efficiency has been set to the value $\epsilon = 0.1$. The comoving rates of black hole accretion have been scaled up by a factor of 3,300 to facilitate visual comparison to the star formation history.

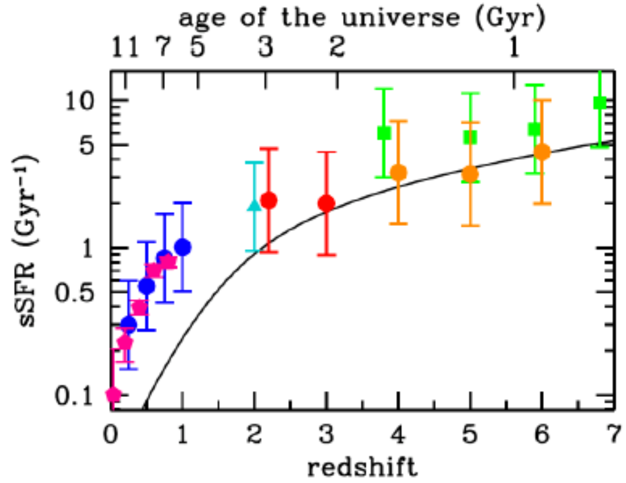
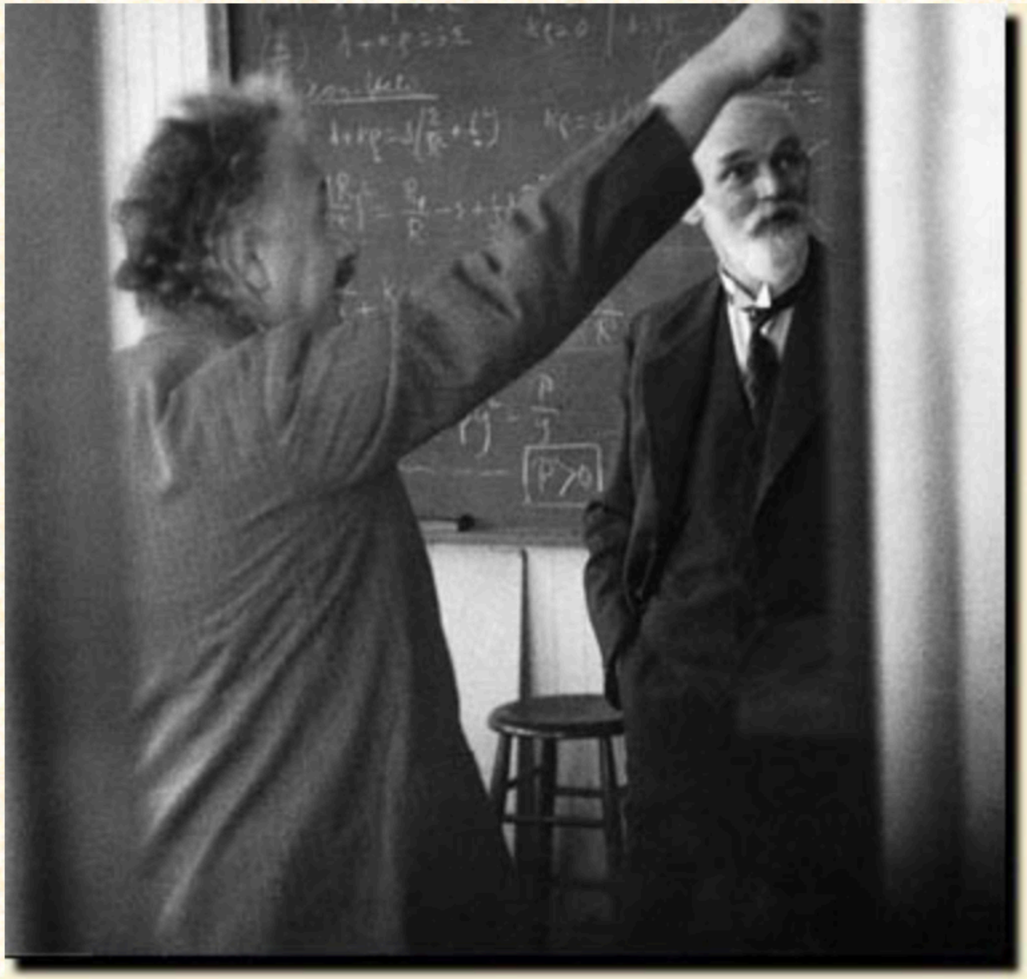


Figure 13: The mean specific star formation rate (sSFR$\equiv$ SFR$/M_*$) for galaxies with estimated stellar masses in the range $10^{9.4} - 10^{10}$ $M_\odot$. The values are taken from the literature: Damen et al. (2009) (*magenta pentagons*), Noeske et al. (2007) (*blue dots*), Daddi et al. (2007) (*cyan triangle*), Reddy et al. (2012) (*red dots*), Stark et al. (2013) (*green squares*), and González et al. (2012) (*orange dots*). The error bars correspond to systematic uncertainties. The González et al. (2012) and Stark et al. (2013) high-redshift points have been corrected upwards for the effect of optical emission lines on the derived stellar masses, using their "RSF with emission lines" model (González et al. 2012) and "fixed Hα EW" model (Stark et al. 2013). The curve shows the predictions from our best-fit SFH.

Einstein – de Sitter universe



Einstein and the astronomer Willem de Sitter in 1932, discussing the equations that provide the best available language for describing the universe seen as a whole.

Equations: how do we compute ages, etc.
(to get some confidence, start with empty case)

In a spatially homogeneous and isotropic universe, the relation among the energy density $\varepsilon(t)$, the pressure $P(t)$, and the scale factor $a(t)$ is given by the Friedmann equation,

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}\varepsilon - \frac{\kappa c^2}{R_0^2 a^2} ,$$

the fluid equation,

$$\dot{\varepsilon} + 3\frac{\dot{a}}{a}(\varepsilon + P) = 0 ,$$

and the equation of state,

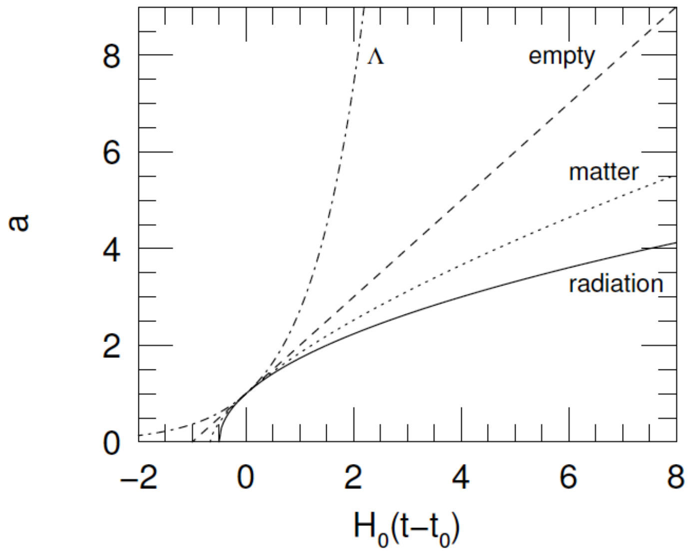
$$P = w\varepsilon .$$

These are what we solve for *P, a, ε*

What about an empty Universe?
(the algebra is really easy for all quantities)

$$\dot{a}^2 = -\frac{\kappa c^2}{R_0^2} \ .$$

One solution to this equation has $\dot{a} = 0$ and $\kappa = 0$. An empty, static, spatially flat universe is a permissible solution to the Friedmann equation.



$$\dot{a} = \pm \frac{c}{R_0} \ .$$

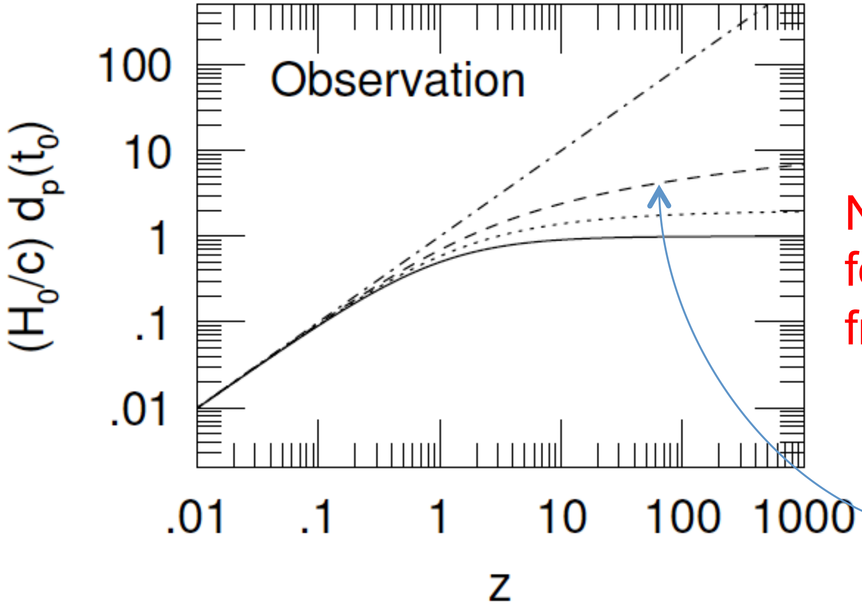
$$a(t) = \frac{t}{t_0}$$

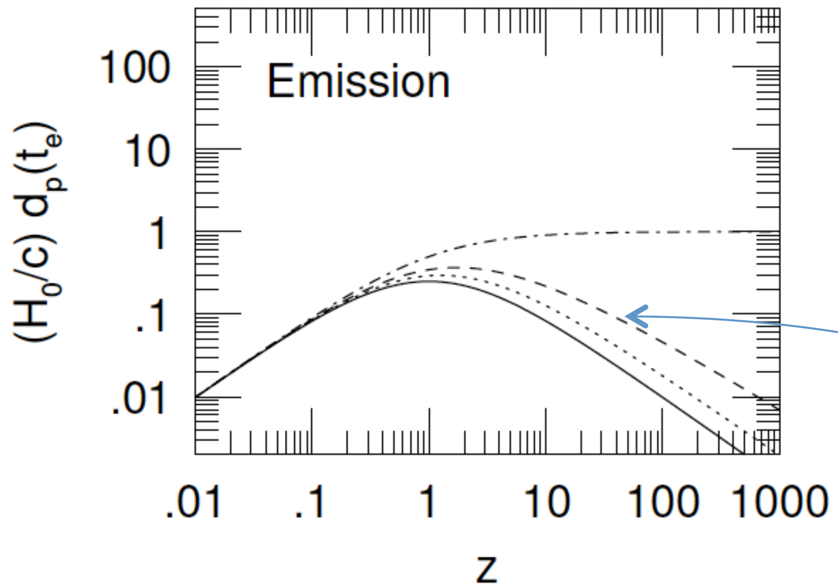
$$t_0 = H_0^{-1} = R_0/c.$$

Empty $\kappa < 0$ Universe (Milne Universe) that can expand or contract

$$1 + z = \frac{1}{a(t_e)} = \frac{t_0}{t_e} \quad \rightarrow \quad t_e = \frac{t_0}{1+z} = \frac{H_0^{-1}}{1+z}$$

What about an empty Universe?
(the algebra is really easy for all quantities)





$$d_p(t_0) = a(t_0)\int_0^r dr = r\ .$$

Null geodesic followed by light from then till now

$$c\int_{t_e}^{t_0}\frac{dt}{a(t)} = \int_0^r dr = r\ .$$

$$d_p(t_0) = c\int_{t_e}^{t_0}\frac{dt}{a(t)}\ .$$

$$d_p(t_0) = ct_0\int_{t_e}^{t_0}\frac{dt}{t} = ct_0\ln(t_0/t_e)$$

$$d_p(t_0) = \frac{c}{H_0}\ln(1+z)\ .$$

$$d_p(t_e) = \frac{c}{H_0}\frac{\ln(1+z)}{1+z}$$

AMAZING CONCLUSION:

We are revisiting the CMB discussion from a few lectures ago.

Objects with much higher redshifts are seen as they were very early in the history of the Universe, when their **proper distance** $d_P(t_e)$ from the observer was very small. The photon's geodesic is moving through a space-time that is being stretched rapidly.

e.g. CMB

$d_P(t_e) = 13$ Mpc
$d_P(t_0) = 13{,}000$ Mpc

The next easiest is a spatially flat ($\kappa = 0$), single component (w) Universe

$$\dot{a}^2 = \frac{8\pi G \varepsilon_0}{3c^2} a^{-(1+3w)}$$

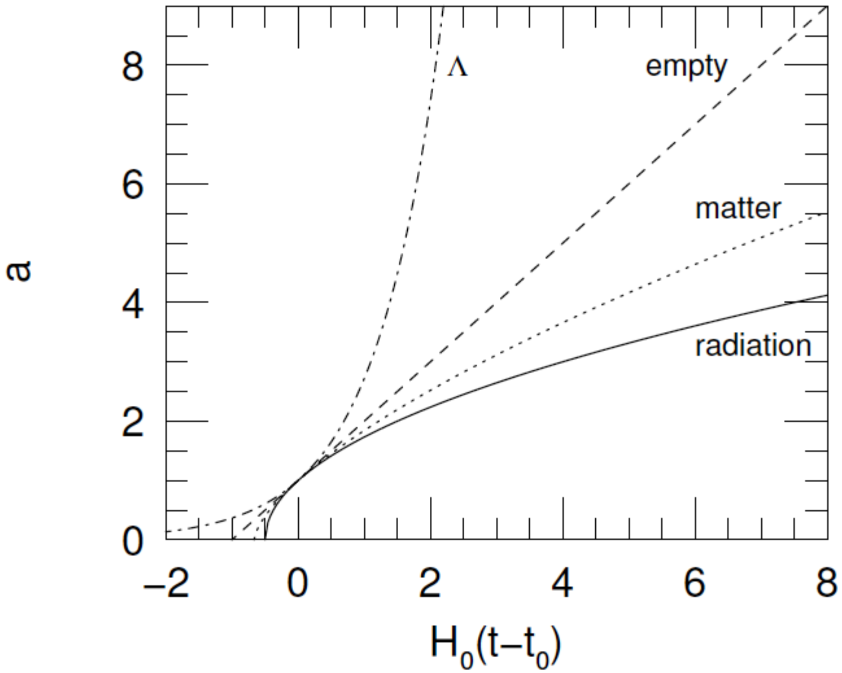
Easy to show solution ($w \neq -1$) $a(t) = (t/t_0)^{2/(3+3w)}$

$$t_0 = \frac{1}{1+w}\left(\frac{c^2}{6\pi G \varepsilon_0}\right)^{1/2}$$

These all follow from simple substitution

$$H_0 \equiv \left(\frac{\dot{a}}{a}\right)_{t=t_0} = \frac{2}{3(1+w)} t_0^{-1}$$

$$t_0 = \frac{2}{3(1+w)} H_0^{-1}$$



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Einstein - de Sitter Universe
Non-relativistic matter, *w*=0

$$t_0 = \frac{2}{3H_0}$$

$$d_{\rm hor}(t_0) = 3ct_0 = 2c/H_0$$

$$a_m(t) = (t/t_0)^{2/3}$$

Radiation, *w*=+1/3

$$t_0 = \frac{1}{2H_0}$$

$$d_{\rm hor}(t_0) = 2ct_0 = c/H_0$$

$$a(t) = (t/t_0)^{1/2}$$

Hi Joss,

I am a PHYS2913 student and I just have a question about the age of the universe t0 in the de-Sitter universe. I saw on tutorial 2 that it is proportional to 1/H0. I am trying to arrive at d_H(t0) = 3.2c/H0 using t0, and for that I've been trying to find the full equation of t0. Could I please have some guidance on how to derive this?

Thank you!

For completeness

w = 0

$$d_p(t_0) = c\int_{t_e}^{t_0} \frac{dt}{(t/t_0)^{2/3}} = 3ct_0[1-(t_e/t_0)^{1/3}] = \frac{2c}{H_0}\left[1-\frac{1}{\sqrt{1+z}}\right]$$

$$d_p(t_e) = \frac{2c}{H_0(1+z)}\left[1-\frac{1}{\sqrt{1+z}}\right]$$

w = +1/3

$$d_p(t_0) = c\int_{t_e}^{t_0} \frac{dt}{(t/t_0)^{1/2}} = 2ct_0[1-(t_e/t_0)^{1/2}] = \frac{c}{H_0}\left[1-\frac{1}{1+z}\right]$$

$$d_p(t_e) = \frac{c}{H_0(1+z)}\left[1-\frac{1}{1+z}\right] = \frac{c}{H_0}\frac{z}{(1+z)^2}$$

The next easiest is a spatially flat ($\kappa = 0$), single component (w) Universe, Lambda case

Constant with time

$$\dot{a}^2 = \frac{8\pi G \varepsilon_\Lambda}{3c^2} a^2$$

$$\dot{a} = H_0 a \qquad H_0 = \left(\frac{8\pi G \varepsilon_\Lambda}{3c^2}\right)^{1/2}$$

$$a(t) = e^{H_0(t-t_0)}$$

Recall we covered the age of a Lambda Universe in an earlier lecture, where t_0 = 0.955/H_0 = 13.74 Gyr (H_0 = 68 km/s/Mpc), $d_H(t_0)$ = 3.20c/H_0 = 3.35ct_0 = 14000 Mpc (student question).

$$d_p(t_0) = c\int_{t_e}^{t_0} e^{H_0(t_0-t)}dt = \frac{c}{H_0}[e^{H_0(t_0-t_e)} - 1] = \frac{c}{H_0}z$$

$$d_p(t_e) = \frac{c}{H_0}\frac{z}{1+z}$$

The most general case with mixed models: you just get long, complicated expressions.

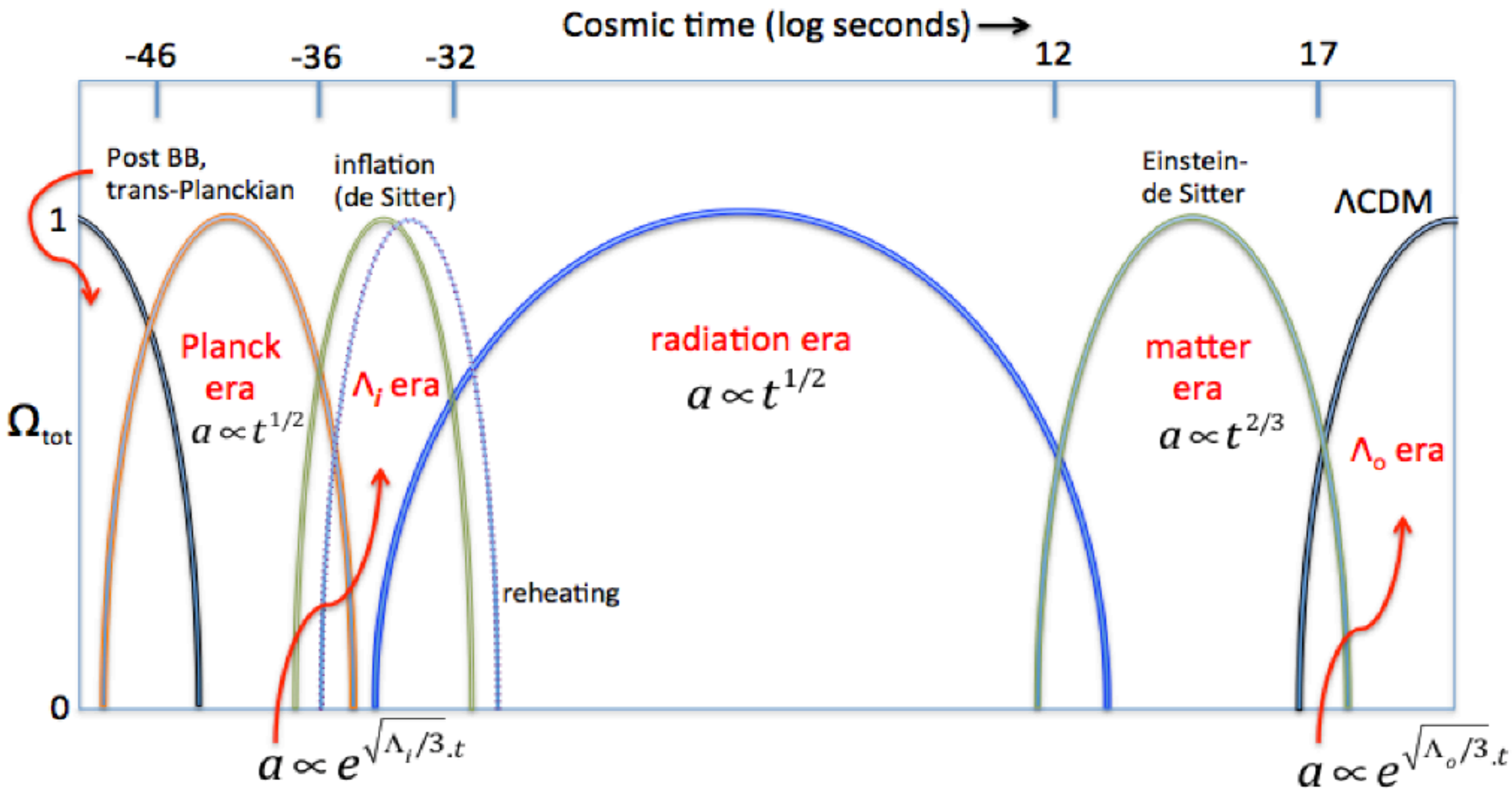
$$\frac{H^2}{H_0^2} = \frac{\Omega_{r,0}}{a^4} + \frac{\Omega_{m,0}}{a^3} + \Omega_{\Lambda,0} + \frac{1 - \Omega_0}{a^2}$$

Curvature term

You can get to most equations from here…

Overview & revision

The Universe is a remarkable study that begins with physics we don't understand (trans-Planckian), moving through QFT (inflation), GR, particle physics, leptogenesis, baryogenesis, and astrophysics once matter takes over.



Remember that the Hubble parameter $H = \frac{\dot{a}}{a}$ so this has a different dependence in each era, declining overall like everything else, e.g. energy/matter (non-Λ) density declining like a^{-3}, temperature, pressure, etc. Note that during the Λ eras, H is a constant. Expanding universes have changing a but not necessarily H.