Conventional Point Cusp Theories Applied to the Polywell

Magnetic Field Structure | Point Cusp Theories | Confinement

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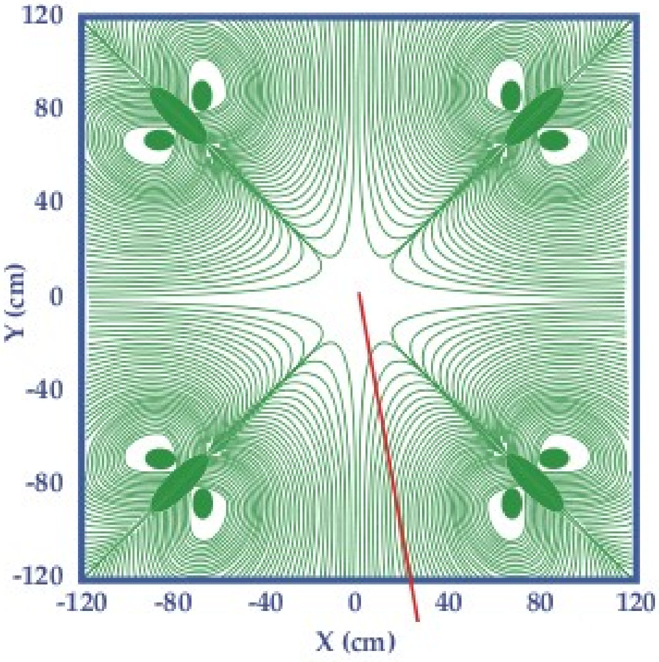
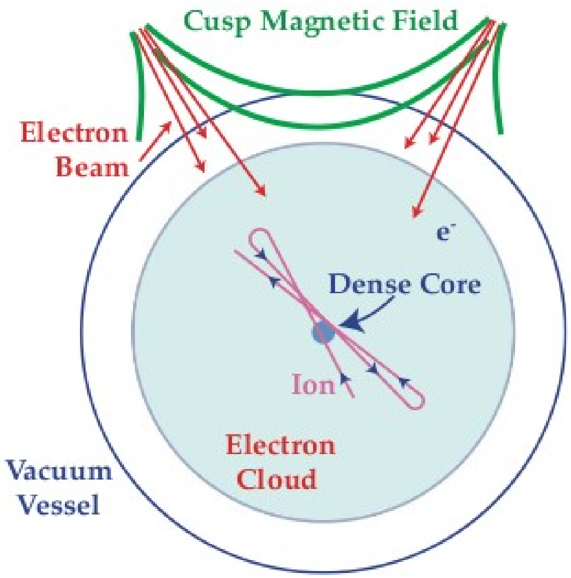
Talk Outline

- The Polywell as a hybrid device
  - IEC embedded in a magnetic field
- Magnetic field structure
  - Electron motion and confinement.
  - Identify two classes of trajectories
  - Compare point and line cusps
- Principle finding
  - At small coil spacings, the Polywell field can be approximated as a system of point cusps.
  - This simplification allows the application of conventional point cusp theories to confinement time calculations
- Limitations → low beta VS high beta operation

The Polywell Concept – A hybrid device

- Uses large magnetic fields in addition to electrostatic grids to create a virtual cathode.
- Field created by pairs of opposing current loops, each creating a cusp about the origin.
- Magnetic fields vanish in centre due to symmetry creating a null point.

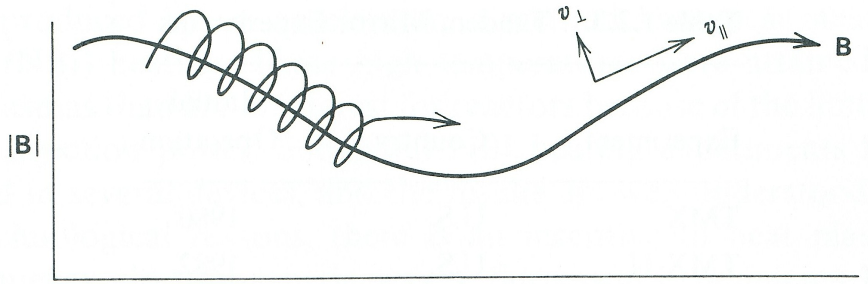


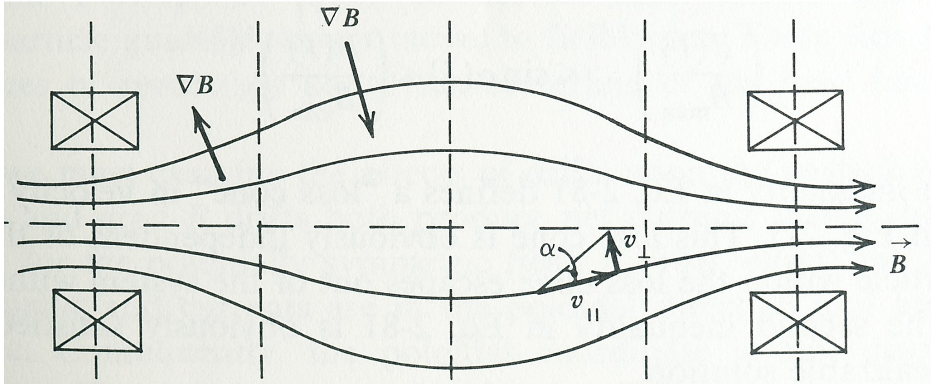


Images: [1] Krall (1992)

Magnetic Mirror Effect

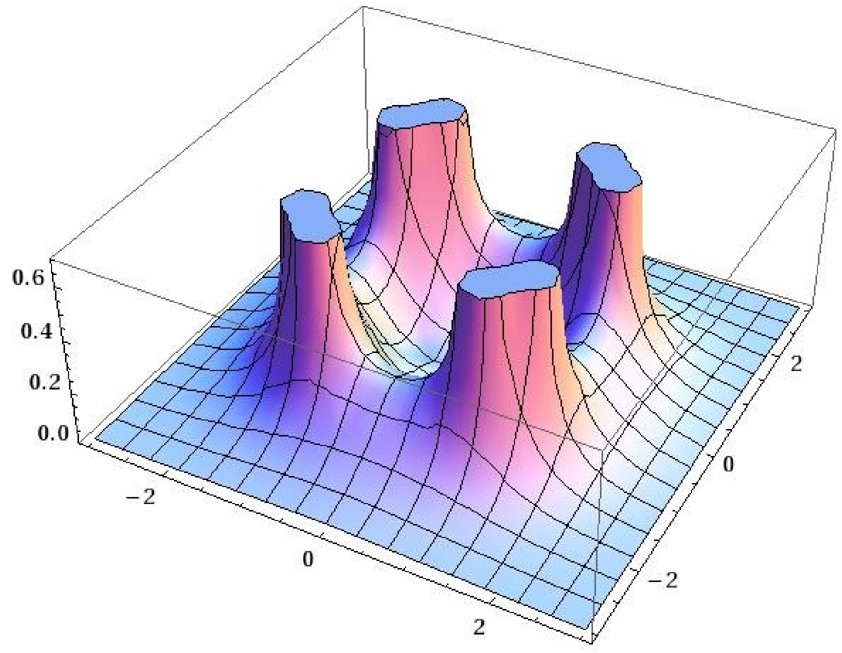
Mirror Machines





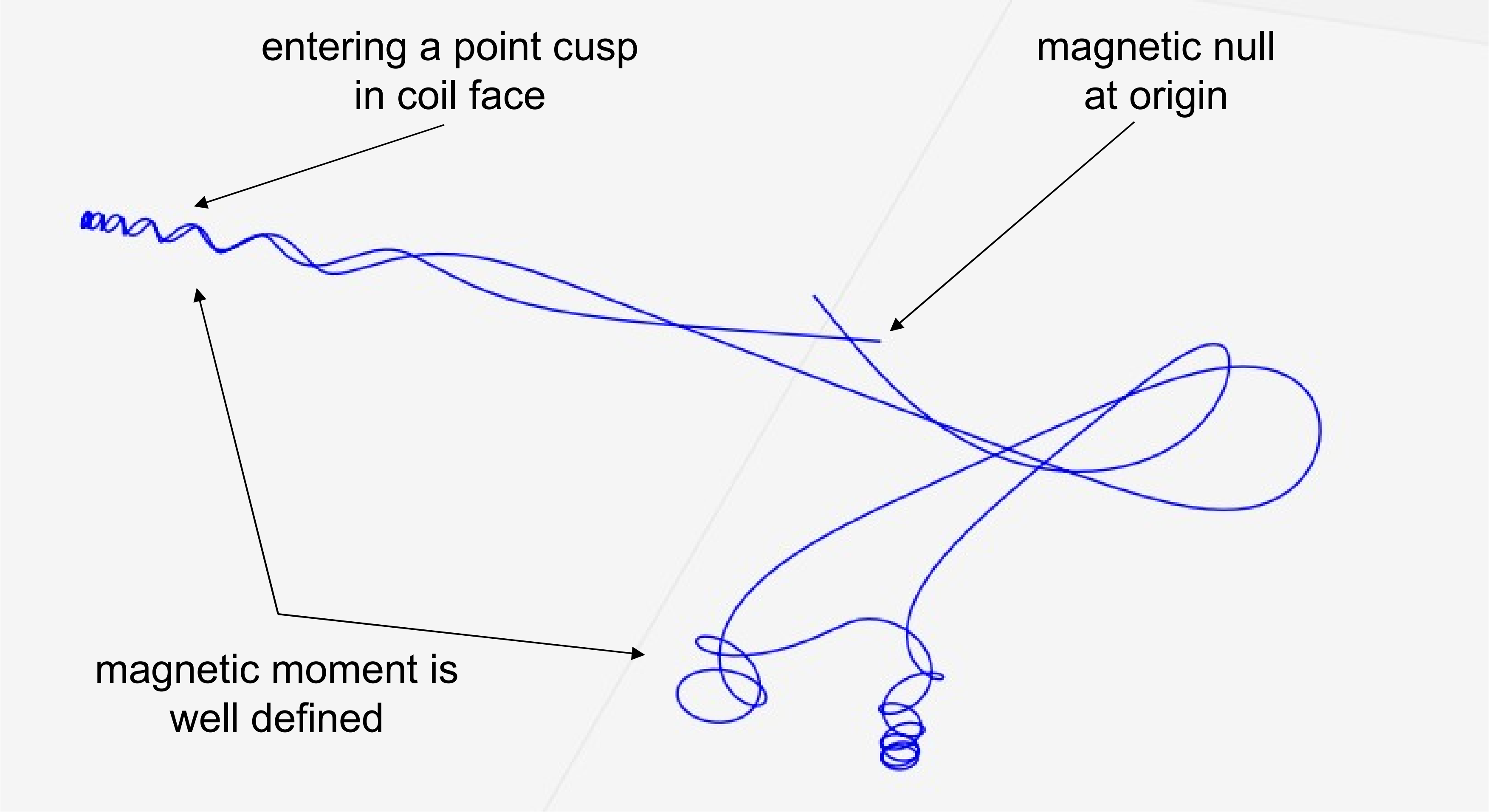
Simple-mirror field configuration.

Absolute Magnitude of B



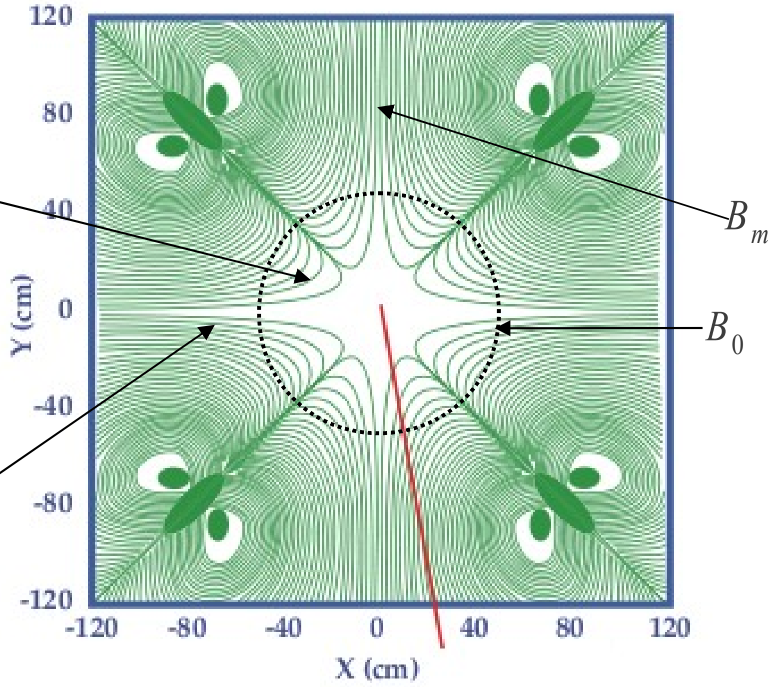
Images: [2] Stacey (1992)

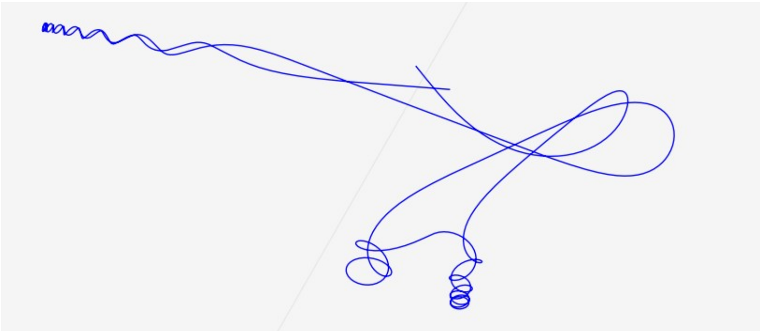
Single Electron Trajectory Simulation



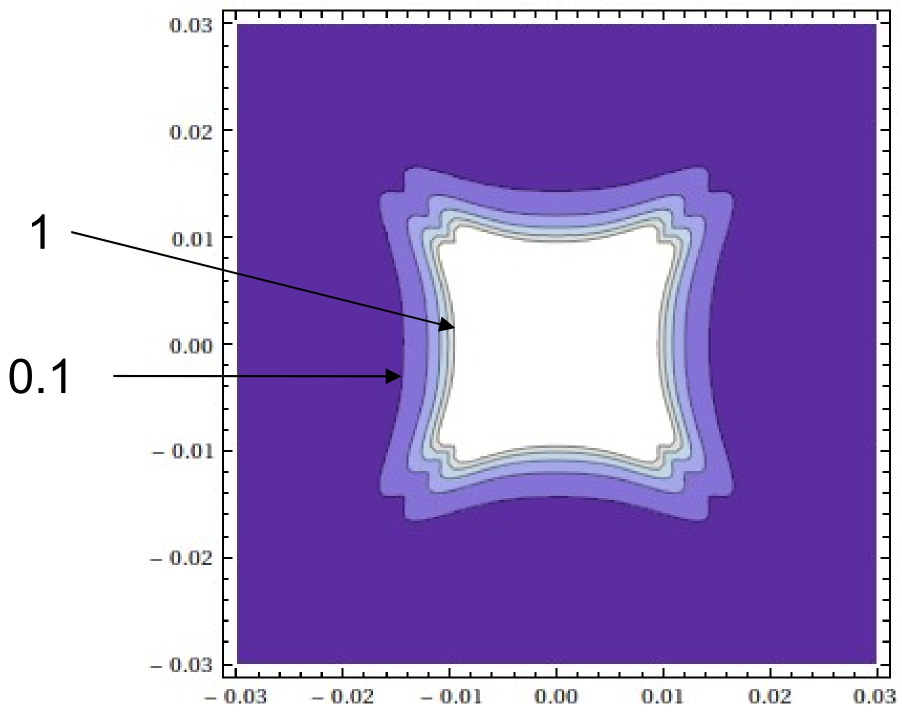
Adiabatic Condition

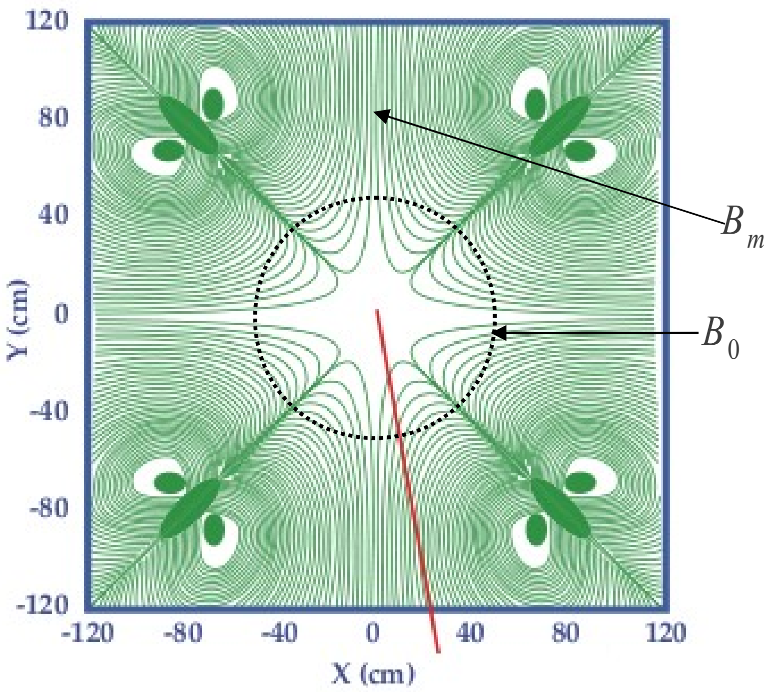
- The electron gyroradius changes rapidly near the centre. Over a distance of ½R it can change from ≈ 1cm to ∞. Thus the magnetic moment μ is not conserved in this region.
- Outside the dashed circle, the gyroradius changes very slowly and the magnetic moment is a constant of motion. In this region the motion is completely adiabatic and the mirror effect applies.
- Reflection condition: $B_m > \frac{KE}{\mu}$





Adiabatic Condition

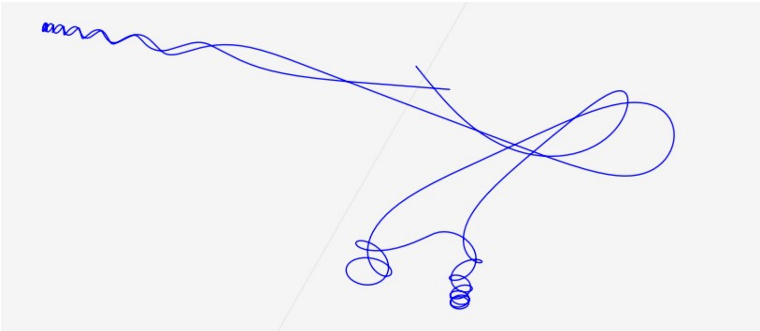




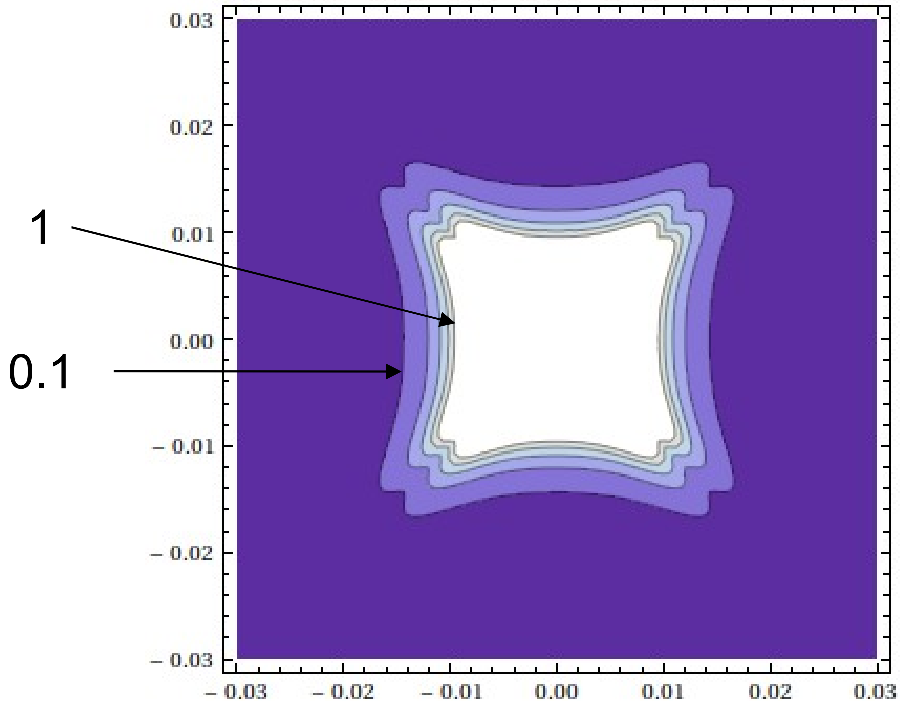
condition for adiabatic μ

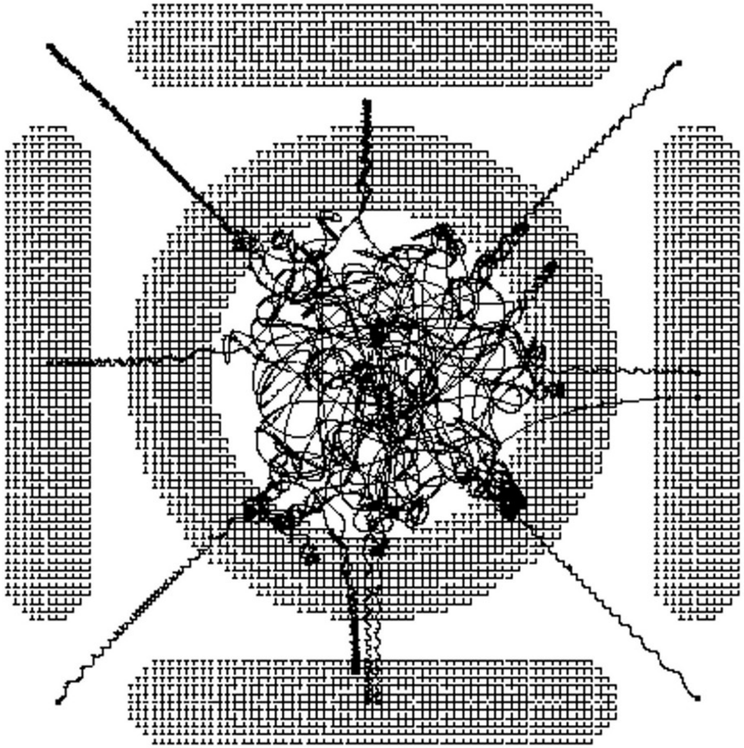
$$r_L \left|\frac{\nabla B}{B}\right| \ll 1$$

(the spatial change in magnetic field occurs slowly compared with the gyroradius)



Adiabatic Condition





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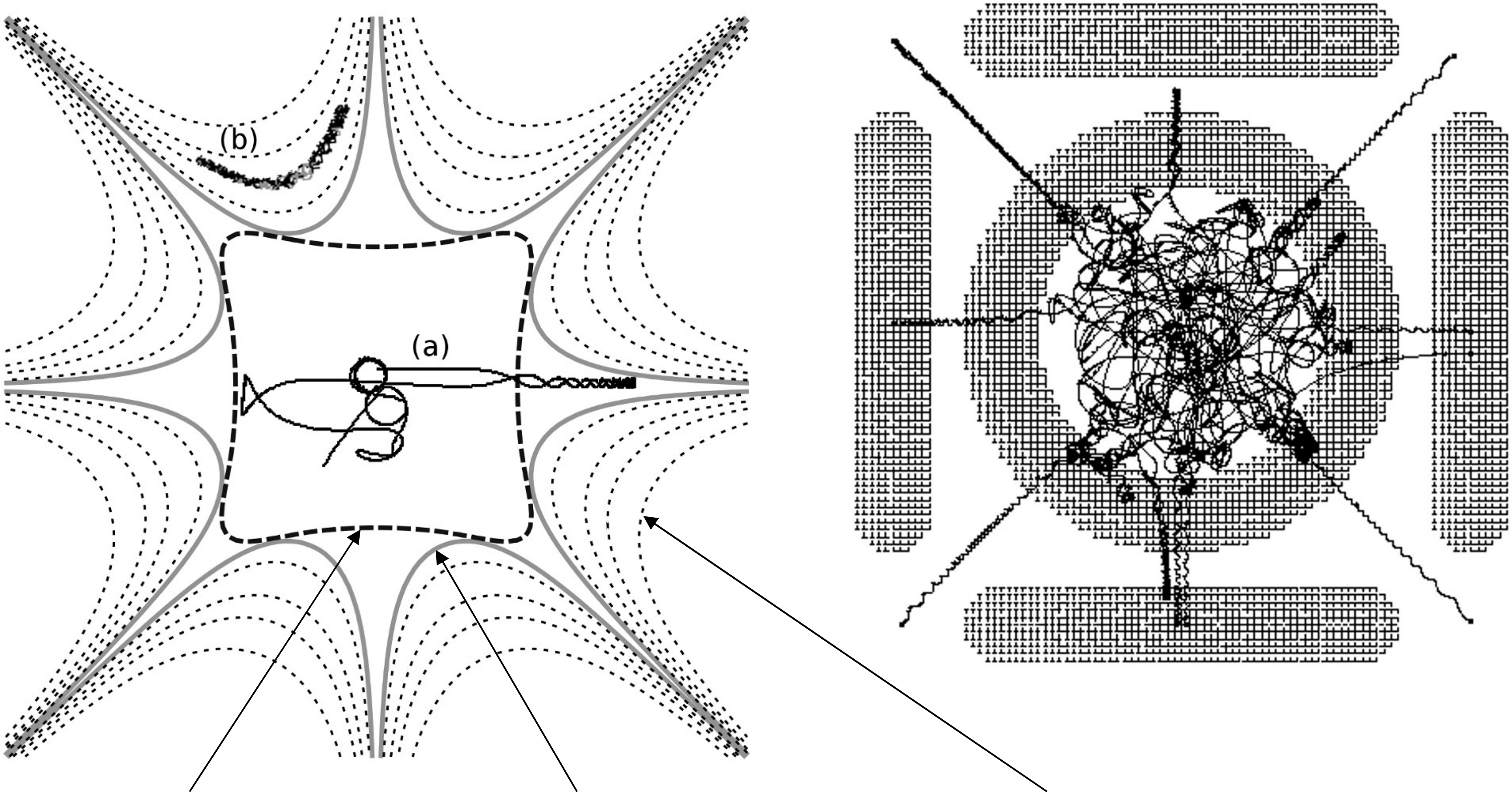
Critical Flux Tube

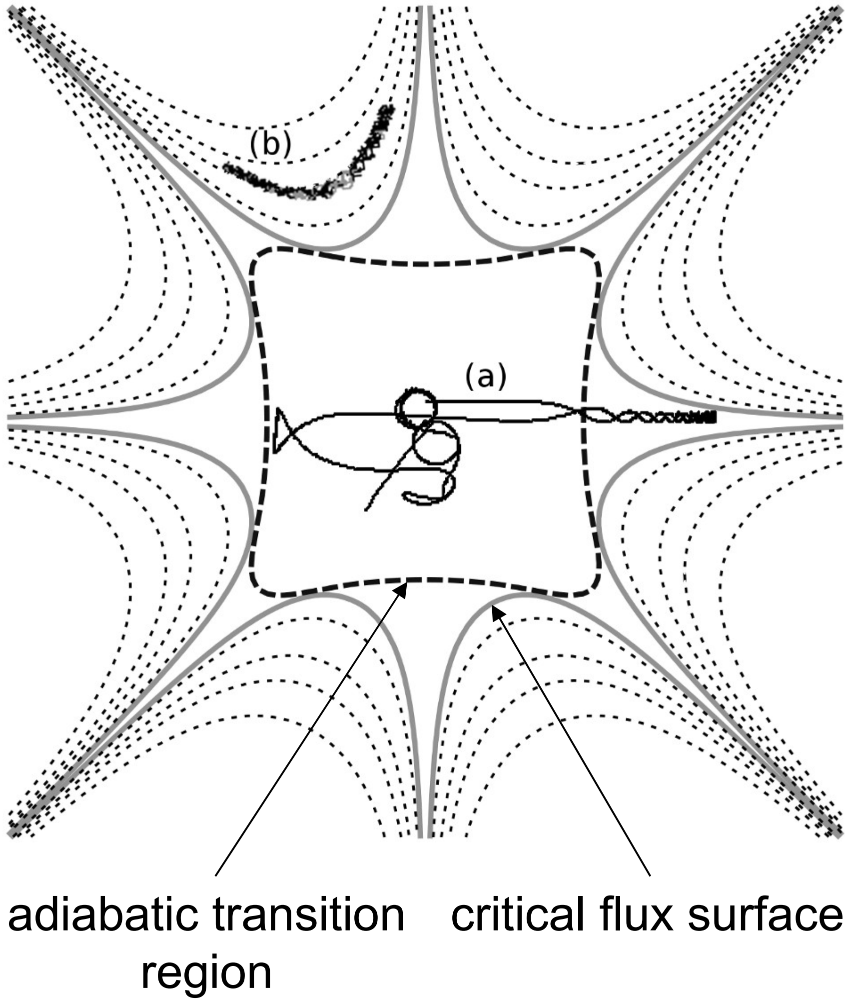


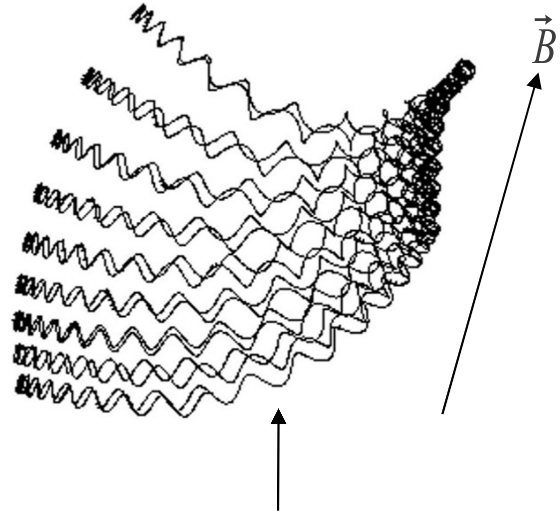
adiabatic transition region

critical flux surface

magnetic fieldlines

Second Class of Trajectories



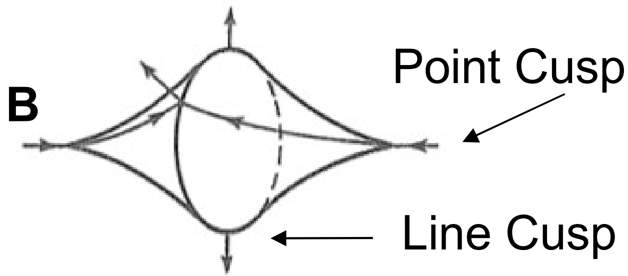


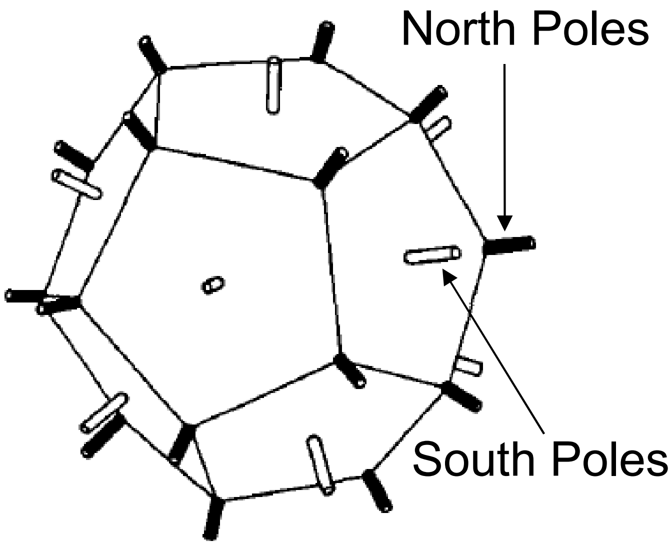
- A 3D isometric view of trajectory (b)
- This motion is completely adiabatic and the electron will be confined indefinitely.
- These trajectories are not suitable for IEC operation

Point and Line Cusps

- Despite MHD stability – rapid plasma loss from the line cusp region. Plugging mechanisms such as RF power and repeller plates have failed to make it work.
- Sadowski (1970) developed a spherical multipole configuration with 30 point cusps [3].
- Central idea is that a system of point cusps will be much more efficient than any system with broad line cusps.
- Observed a confinement time 2.5 times longer than a spindle cusp.
- NOTE: Sadowski did not do IEC, only neutral plasma confinement.

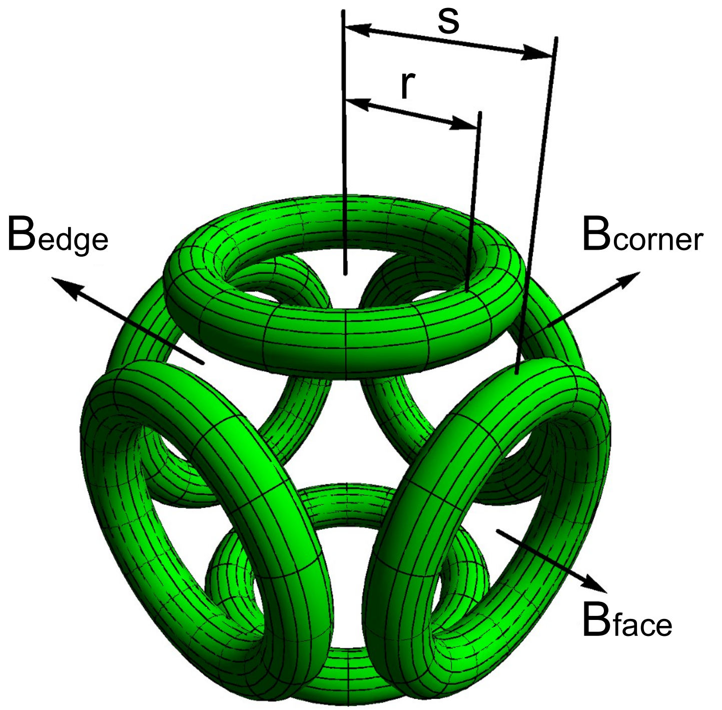
Spindle Cusp Device



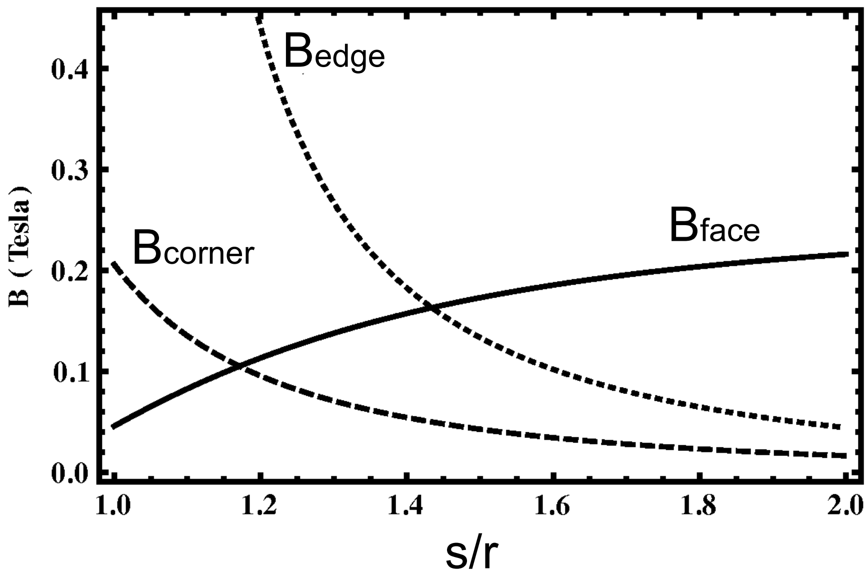


Sadowski's dodecahedron [3]

Point and Line Cusps



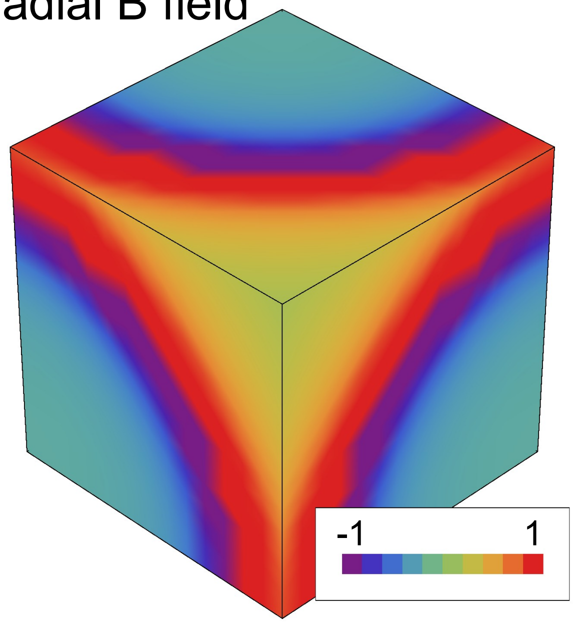
B field as a function of coil spacing



- At small spacings (s/r → 1) B face is extremely weak. But at large spacings (s/r → 2) B corner is very weak.
- The ideal spacing is approx 1.2 where Bface = Bcorner and Bedge is an order of magnitude larger than both, effectively plugging the line cusp.

Point and Line Cusps

Sign and relative magnitude of radial B field



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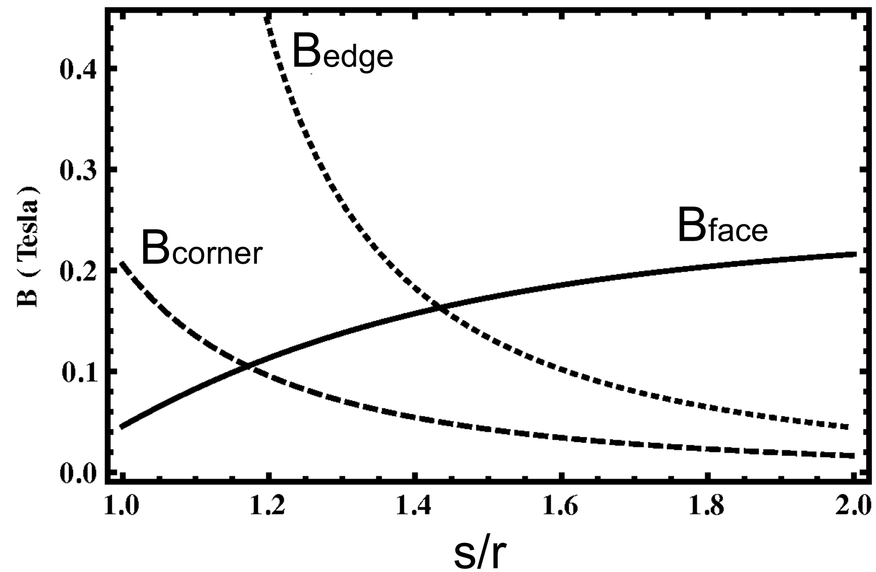

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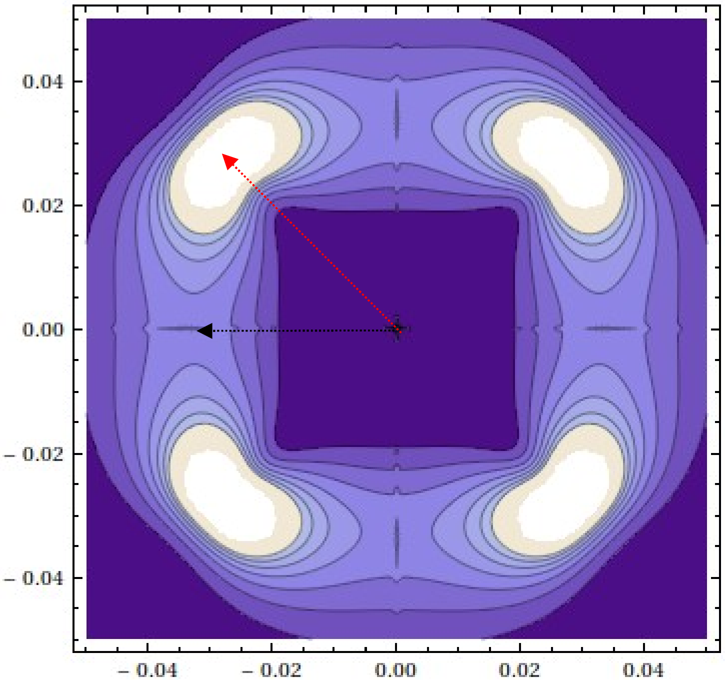
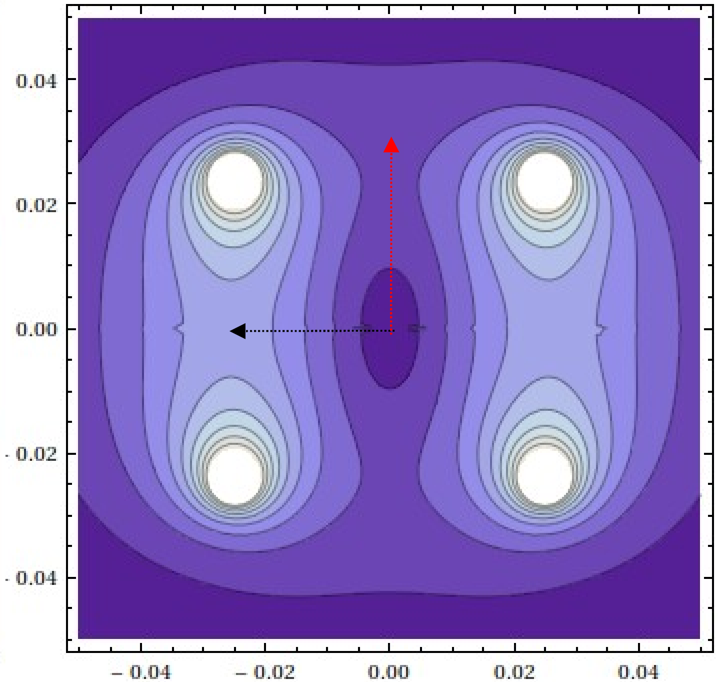
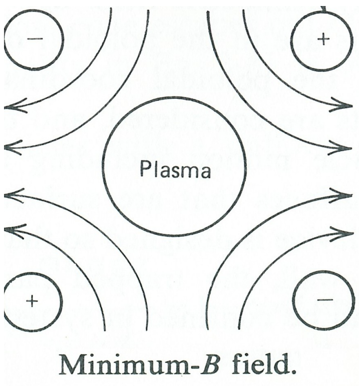
Comparison with Cusps

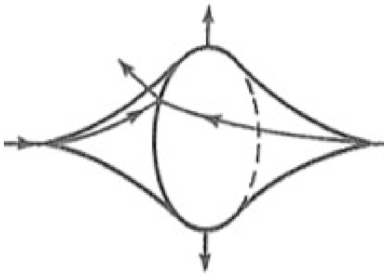
Polywell Field
Magnitude of B

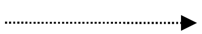


Spindle Cusp Field
Magnitude of B







- Point cusp escape trajectory: 
- Line cusp escape trajectory: 

Approximate Loss Rate

- Fraction of electrons inside the loss cone of a point cusp is well known [2].
- Bussard argued that that if the loss cones are not overlapping, the conventional equation only needs to be modified by a factor n, where n is the number of point cusps [4]. We have shown that n = 14 for a cubic polywell.
- The loss fraction can be interpreted as the probability of escape after each successive random scattering event inside the magnetic null region.

$$L = \frac{n}{2}\left(1 - \sqrt{1 - \frac{B_0}{B_m}}\right)$$

Loss Fraction

$$\frac{dN(t)}{dt} = -\frac{LN}{\tau_{trans}}$$

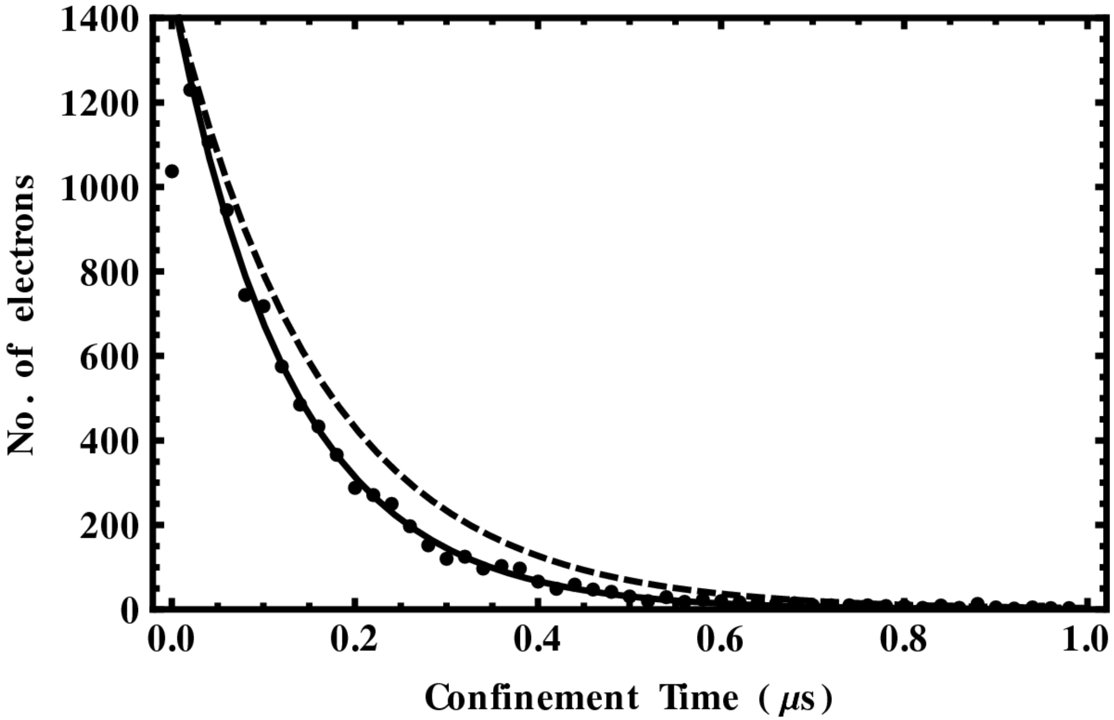
$$\therefore N(t) = N_0 e^{\frac{-Lt}{\tau_{trans}}}$$

Loss Rate

$$\tau_{trans} = \frac{2}{v_0}\int \sqrt{1 - \frac{B(z)}{B_0}\sin^2\theta_0 dz} + 2\frac{10r_0}{v_0}$$

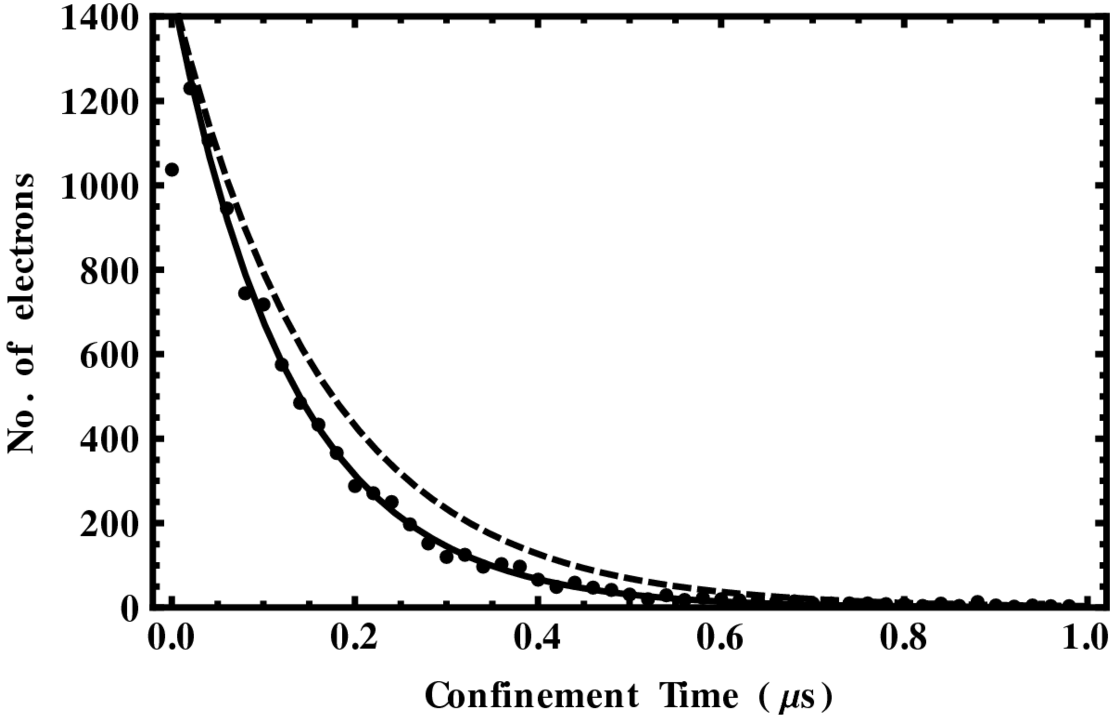
Transit Time Estimate

Confinement Time



- The distribution of confinement times for 10,000 electrons [5].
- Simulation $\tau_0 = 0.129\mu s$ vs theory $\tau_0 = 0.163\mu s$

Confinement Time



- The distribution of confinement times for 10,000 electrons [5].
- Simulation $\tau_0 = 0.129\mu s$ vs theory $\tau_0 = 0.163\mu s$

If $\tau_0 = 0.15\mu s$, then a litre of 100eV electrons at a density of 10^{19} m^{-3} requires approximately 400kW of input power to replace energy lost by electrons.

Limitations and further questions

- Model only applies to low beta
- Other effects that need to be considered:
  - Need to establish how high beta changes the flux surfaces → effective loss area
  - Low beta confinement times appear too short for efficient IEC operation.
  - Electrostatic plugging of point cusps during high beta may improve confinement times.
  - Modification to include radial electric fields created by virtual cathode
  - Are completely adiabatic orbits unfavorable? How do they effect the potential well?

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References

[1] N. Krall, “The Polywell: A Spherically Convergent Ion Focus Concept". Fusion technology **22** (1992), pp. 42-49.

[2] W. Stacey, “Fusion plasma analysis”. Wiley-Interscience (1981).

[3] M. Sadowski, “Plasma containment in a spherical multipole magnetic trap”. Journal of Plasma Physics **4** (1970), pp. 1-12.

[4] R. Bussard and N. Krall, EMC2 Technical Report 0191-02, 1991

[5] M. Carr et al. “Low beta confinement in a Polywell modelled with conventional point cusp theories". Physics of Plasmas **18** (2011), p. 112501.

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