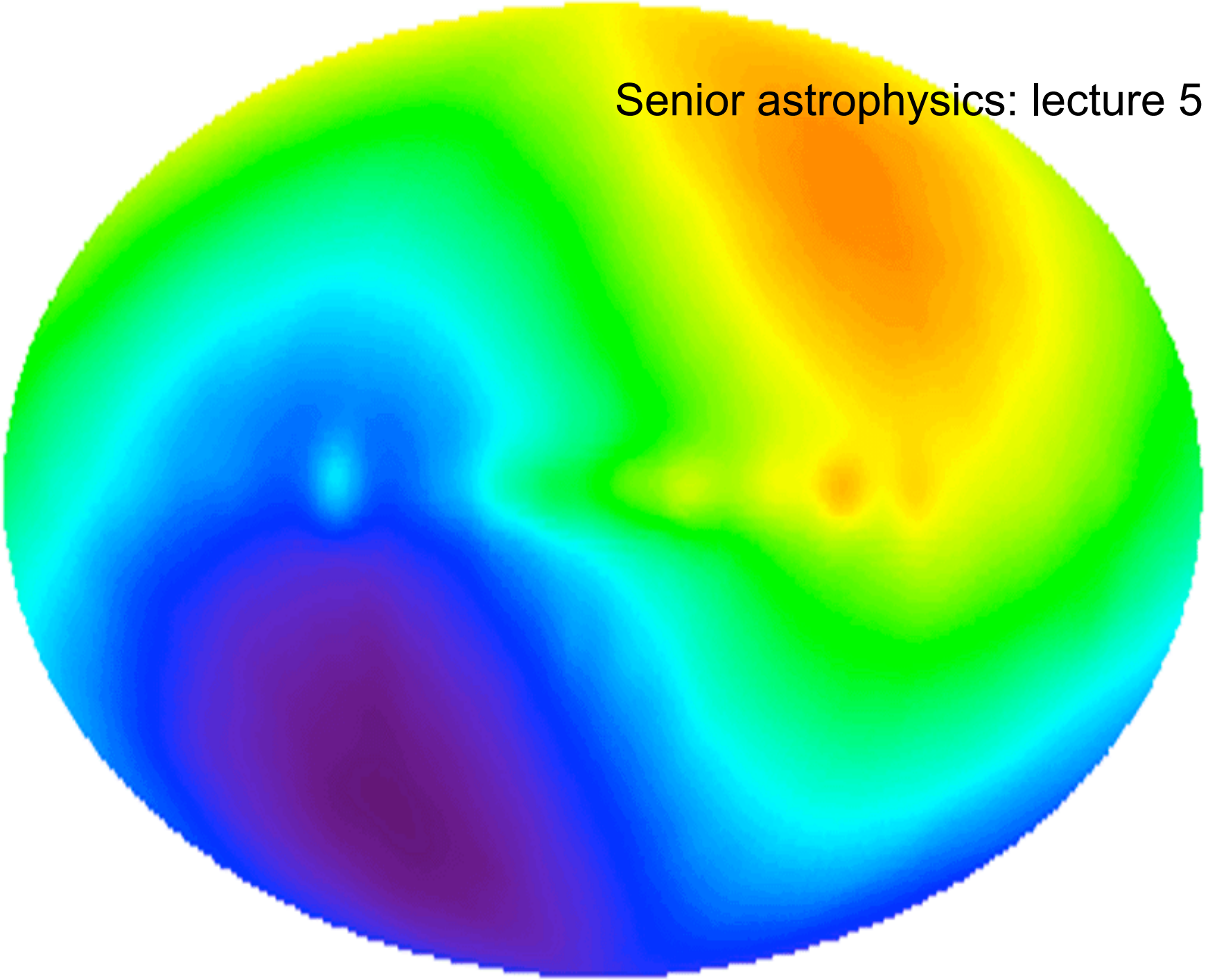
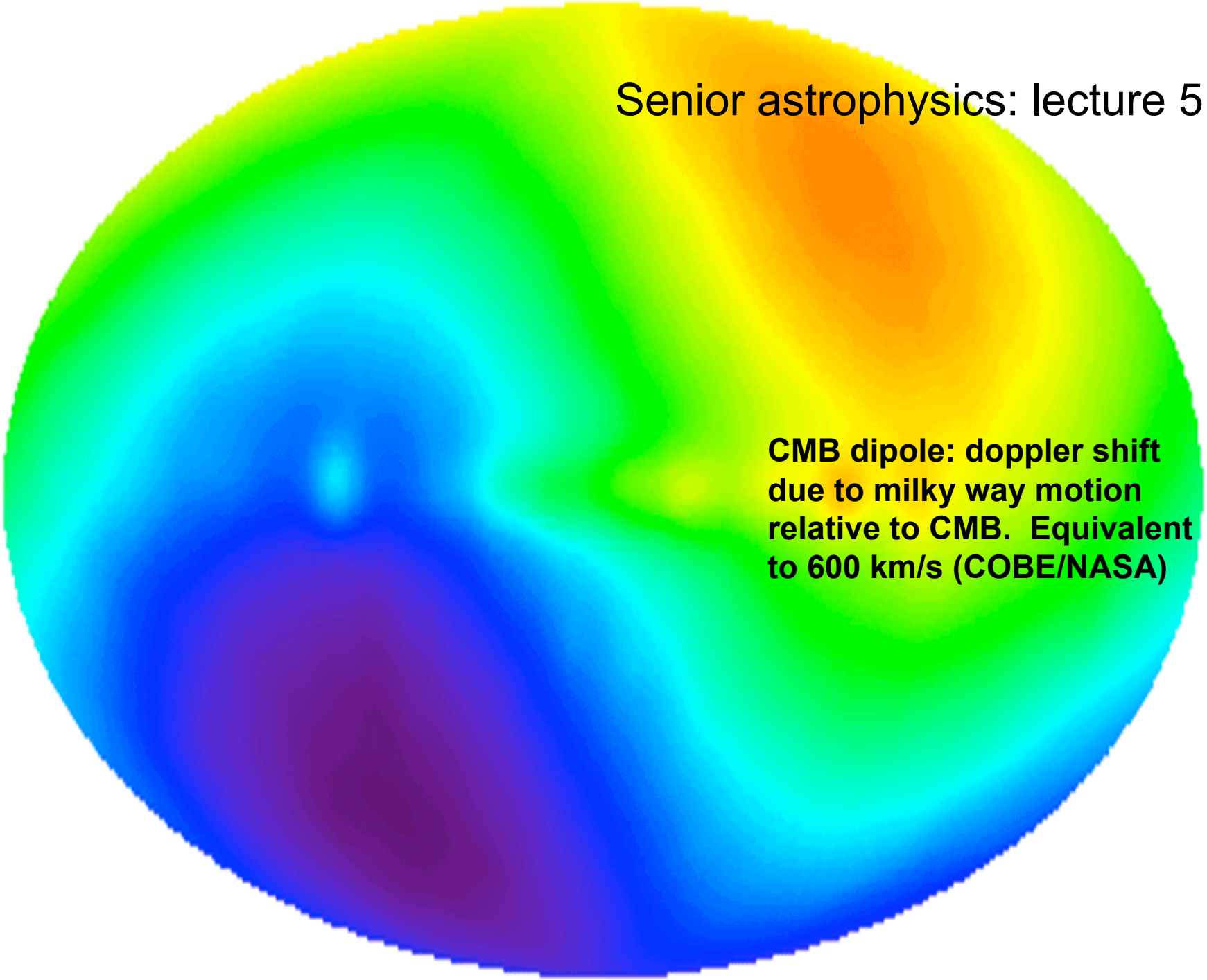


Senior astrophysics: lecture 5



CMB dipole: doppler shift due to milky way motion relative to CMB. Equivalent to 600 km/s (COBE/NASA)

Senior Astrophysics

- Scott Croom: scroom@physics.usyd.edu.au rm 561A
  - Lectures 1 to 9: Cosmology
  - 1st assignment given: 20th September, due 4th October
  - Lecture 9 ADV/REG split (6th Oct).

- Helen Johnston: helenj@physics.usyd.edu.au rm 563
  - Lecture 10 to 19: Stellar evolution and the end states of stars.
  - 2nd assignment given: 13th Oct, due 27th October
  - Lecture 19 ADV/REG split.

Lecture 5: distances and the CMB

Revision from lecture 4: Solving the Friedmann equations for simple model Universes.

1. Comoving distance.
2. Horizons.
3. Angular diameter distance, luminosity distance and surface brightness.
4. Black body distributions.
5. The cosmic microwave background.
6. Observations of the CMB.

Wikipedia: Comoving distance, luminosity distance, angular diameter distance, cosmic microwave background

Books: Liddle: Ch 10 + Adv. Topic 2

Revision: density parameters

› Friedmann equation, assuming flatness leads to critical density:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} \Rightarrow \rho_{crit}(t) = \frac{3H^2}{8\pi G}$$

$$\Omega(t) = \frac{\rho}{\rho_{crit}}$$

$$\rho_{crit}(t_0) = \frac{3H_0^2}{8\pi G} \approx 1.88h^2 \times 10^{-26} kg/m^3$$

If $\rho > \rho_{crit}$ k is $+$ve Universe is closed (will recollapse).

If $\rho < \rho_{crit}$ k is $-$ve Universe is open (will expand forever).

› Density parameters:

$$\Omega_M = \rho_{M,o}\frac{8\pi G}{3H_o^2}, \quad \Omega_R = \rho_{R,o}\frac{8\pi G}{3H_o^2}, \quad \Omega_\Lambda = \frac{\Lambda}{3H_o^2}$$

$$\Omega_K = -\frac{kc^2}{a_o^2 H_o^2}$$

$$\Omega = \Omega_M + \Omega_R + \Omega_\Lambda = 1 - \Omega_K$$

$$1 = \Omega_M + \Omega_R + \Omega_\Lambda + \Omega_K = \text{FRIEDMANN EQUATION TODAY}$$

Revision: simple cosmological models

- Einstein-de Sitter: $\Omega_M=1$, $\Omega_R=0$, $\Omega_\Lambda=0$, i.e., flat (k=0):

$$a(t) = a_0\left(\frac{t}{t_0}\right)^{2/3}, \quad H(t) = \frac{2}{3t}, \quad t_0 \approx 9Gyr$$

- Radiation dominated: $\Omega_M=0$, $\Omega_R=1$, $\Omega_\Lambda=0$, i.e., flat (k=0):

$$a(t) = a_0\left(\frac{t}{t_0}\right)^{1/2}, \quad H(t) = \frac{1}{2t}, \quad t_0 \approx 7Gyr$$

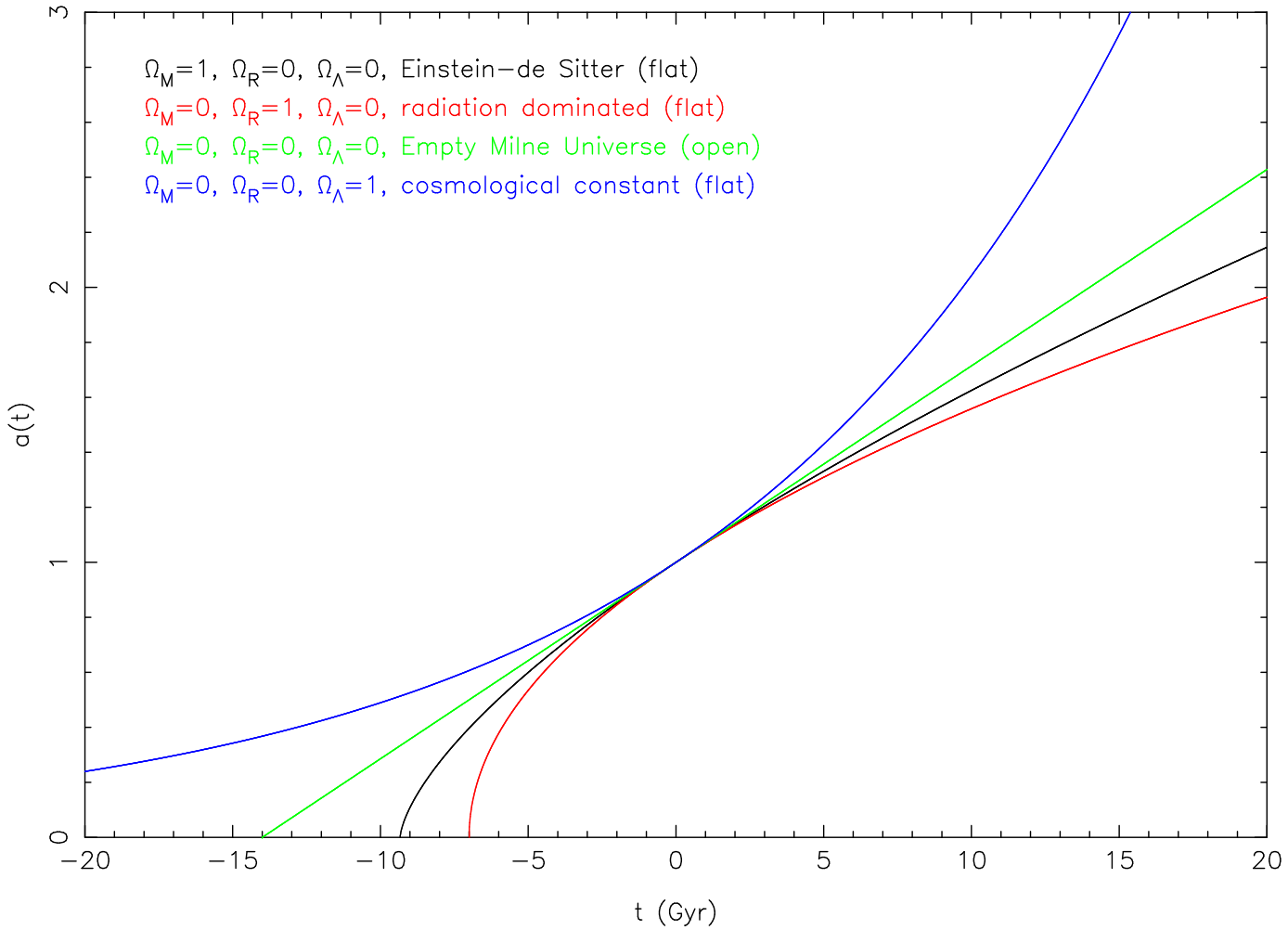
- Empty "Milne" Universe: $\Omega_M=0$, $\Omega_R=0$, $\Omega_\Lambda=0$, i.e., open (k=-1):

$$a(t) = a_0\left(\frac{t}{t_0}\right), \quad H(t) = \frac{1}{t}, \quad t_0 \approx 14Gyr$$

- Cosmological constant: $\Omega_M=0$, $\Omega_R=0$, $\Omega_\Lambda=1$, i.e., flat (k=0):

$$a(t) \propto \exp\left[\left(\frac{\Lambda}{3}\right)^{1/2} t\right] \propto \exp[Ht] \text{ big bang at t = -}\infty$$

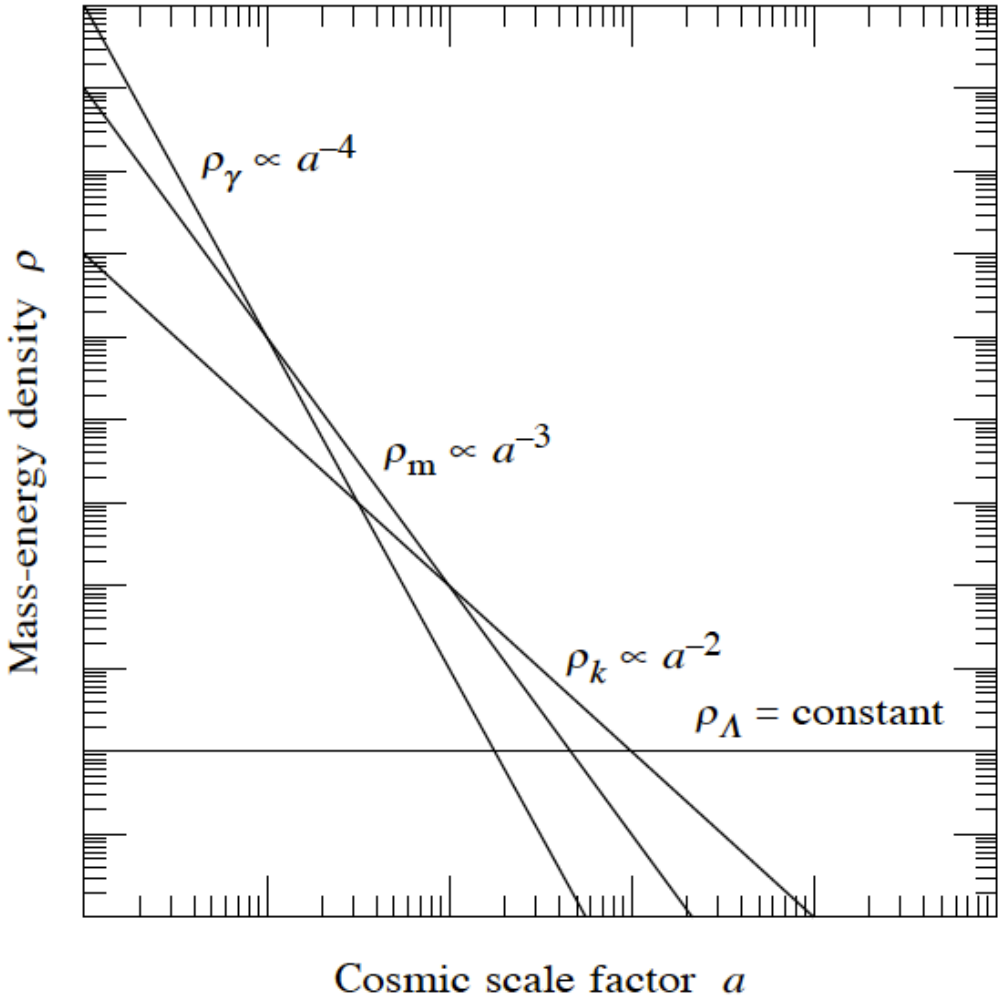
Revision: evolution of the scale factor



› Models matched to the same observed H_0 and a(t)=1 at t=0 (i.e. set current time, t_0=0).

Revision: density evolution of components

- Evolution of density with scale factor:
  - Radiation $\rho \sim a^{-4}$
  - Matter $\rho \sim a^{-3}$
  - Curvature $\rho \sim a^{-2}$
  - Cosmological constant $\rho \sim a^{0}$
- Radiation will always dominate at early times (i.e. small scale factor)
- Cosmological constant (if present) will always dominate at large scale factor.



5.1 Comoving distance

- Q: How do we measure distance in cosmology?
- A: Many different ways – ***depends on the exact question you are asking.***
- Most fundamental is the **comoving distance**. This is r in the FRW metric and normalized to the scale factor now (a_0): **$d_c=a_0 r$**
- Consider an object at a comoving distance $a_0 r_1$:

$$\int_0^{r_1} \frac{dr}{[1-kr^2]^{1/2}} = \int_{t_1}^{t_o} \frac{cdt}{a(t)} = \int_{a_1}^{a_o} \frac{c}{a}\frac{da}{\dot{a}} \qquad \text{as } \dot{a} = da/dt$$

- But observable is redshift, z: $(1+z)=a_0/a$

$$\int_0^{r_1} \frac{dr}{[1-kr^2]^{1/2}} = \frac{c}{H_0 a_0}\int_0^{z} \frac{H_0 dz}{H(z)}$$

5.1 Comoving distance

› Once we have this in terms of the Hubble parameter and redshift, we can substitute the Friedmann equation.

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho_m}{3} + \frac{8\pi G\rho_r}{3} - \frac{kc^2}{a^2} + \frac{\Lambda}{3},\ \rho_m = \rho_{m,0}\left[\frac{a_0}{a}\right]^3 \text{ and } \rho_R = \rho_{R,0}\left[\frac{a_0}{a}\right]^4$$

$$\Omega_m = \rho_{m,0}\frac{8\pi G}{3H_0^2} \text{ and } \Omega_R = \rho_{R,0}\frac{8\pi G}{3H_0^2}$$

$$\Rightarrow \frac{H^2}{H_0^2} = \Omega_m\left(\frac{a_0}{a}\right)^3 + \Omega_R\left(\frac{a_0}{a}\right)^4 - \frac{kc^2}{a^2H_0^2} + \frac{\Lambda}{3H_0^2}$$

$$\text{but } \Omega_k = -\frac{kc^2}{a_0^2H_0^2} \text{ and } \Omega_\Lambda = \rho_\Lambda\frac{8\pi G}{3H_o^2} = \frac{\Lambda}{3H_o^2}$$

$$\text{so } \frac{H^2}{H_0^2} = \Omega_m\left(\frac{a_0}{a}\right)^3 + \Omega_R\left(\frac{a_0}{a}\right)^4 + \Omega_k\left(\frac{a_0}{a}\right)^2 + \Omega_\Lambda$$

$$\text{and } \boxed{\frac{H^2}{H_0^2} = \Omega_m(1+z)^3 + \Omega_R(1+z)^4 + \Omega_k(1+z)^2 + \Omega_\Lambda}$$

Evolution in terms of observable redshift

5.1 Comoving distance

› So in the most general case, we can find the comoving distance by solving:

$$\int_0^{r_1} \frac{dr}{[1-kr^2]^{1/2}} = \frac{c}{H_0 a_0} \int_0^z \frac{dz}{\sqrt{\Omega_m (1+z)^3 + \Omega_R (1+z)^4 + \Omega_k (1+z)^2 + \Omega_\Lambda}}$$

› Unsurprisingly, this is cannot be done analytically (and we won't do it in this course!)

› But making some assumptions about densities and curvature we can get simpler solutions.

› For an Einstein-de Sitter Universe (flat, Ω_M=1, Ω_R=0, Ω_Λ=0):

$$\int_0^{r_1} dr = \frac{c}{H_0 a_0} \int_0^z \frac{dz}{\sqrt{(1+z)^3}}$$

5.1 Comoving distance

› Then we have:

$$r_1 a_0 = \frac{c}{H_0}\int_0^z \frac{dz}{(1+z)^{3/2}} = \frac{c}{H_0}\left[\frac{-2}{(1+z)^{1/2}}\right]_0^z$$

$$d_c = r_1 a_0 = \frac{2c}{H_0}\left[1 - \frac{1}{(1+z)^{1/2}}\right]$$

› This solution was used for most of the 20th century.

› How far can we see?

as $z \to \infty$ we can define a horizon distance $d_H = \frac{2c}{H_0}$

Recalling that $H_0 = \frac{2}{3t_0}$ for an Einstein - de Sitter Universe we have

$$d_H = 3ct_0$$

5.2 The particle horizon

› In an EdS Universe we have:

$$d_H(t_0) = r(z \to \infty)a_0 = \frac{2c}{H_0} = 3ct_0 \text{ and at any time } d_H(t) = r(z \to \infty)a(t) = 3ct$$

› This is the furthest ***proper*** (=comoving at t_0) distance that an event can currently be observed and is called the **Particle Horizon**.

› Crucially, this is finite, even as $t \to 0$.

› Infact, recall that

$$a(t) = \left(\frac{t}{t_0}\right)^{2/3}$$

$$\text{so } \frac{d_H(t)}{a(t)} = \frac{3ct}{t^{2/3}} t_0^{-2/3} \text{ and } \frac{d_H(t)}{a(t)} \propto t^{1/3}$$

› The fraction of the Universe that is visible increases with time, and decreases towards the big bang.

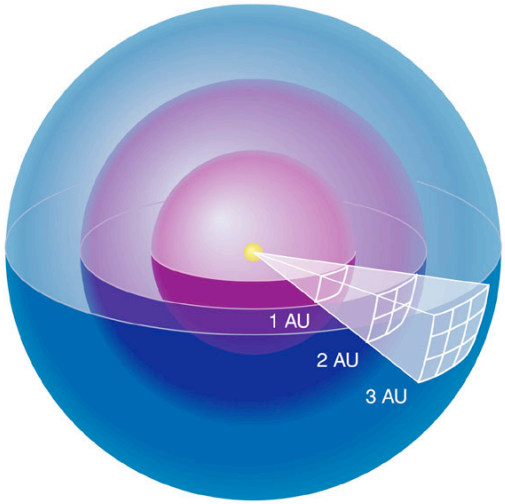
› In general this depends on the content of the Universe.

5.3 Luminosity distance

› Total (bolometric) luminosity (units: W): $L = \frac{Nh\nu_e}{\Delta t_e}$

› area of photon sphere (inverse square law): $A = 4\pi d_c{}^2 = 4\pi\left(a_0 r_1\right)^2$

› redshift: $\lambda_o = \lambda_e\,(1+z) \Rightarrow \nu_o = \nu_e \big/ (1+z)$



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› time dilation: lower photon arrival rate: $\Delta t_o = \Delta t_e\,(1+z)$

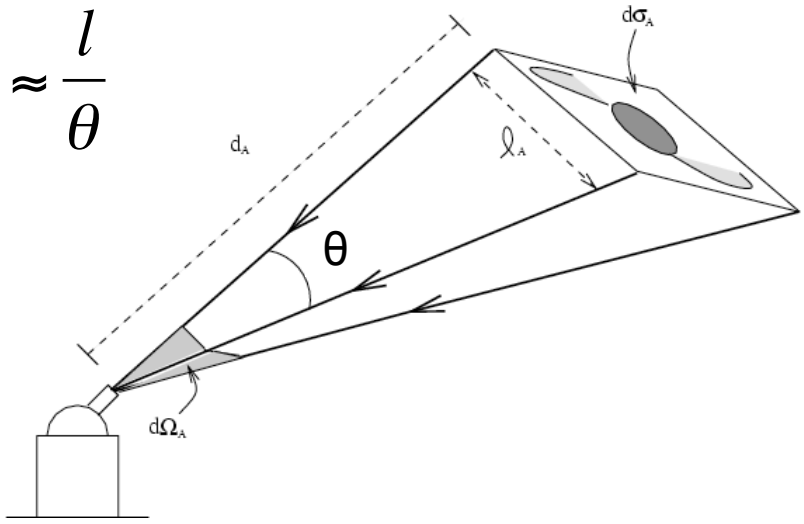
› observed flux (W m^{-2}) $F = \frac{Nh\nu_o}{A\,\Delta t_o} = \frac{L}{4\pi\, d_c^2\,(1+z)^2} = \frac{L}{4\pi\, d_L^2}$

› **Luminosity Distance**: $d_L = d_c(1+z)$

Sources look fainter.

5.3 Angular diameter distance

$$d_A \approx \frac{l}{\theta}$$



- Consider on object with fixed physical size, l, subtending an angle θ on the sky:
- Define l from the metric (when light emitted):

$$l = ds = r_1 a(t_e) d\theta$$

- Light rays propagate radially towards us, so angular extent is preserved. Our perceived angular size is:

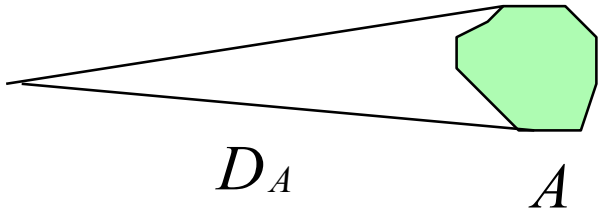
$$d\theta = \frac{l}{r_1 a(t_e)} = \frac{l(1+z)}{r_1 a_0}$$

- Hence we can define the **Angular Diameter Distance:**

$$d_A = \frac{d_c}{(1+z)} = \frac{d_L}{(1+z)^2}$$

5.3 Surface brightness

› Solid angle: $\Omega = \frac{A}{d_A^2}$



› Surface brightness

- Flux per solid angle (e.g. W m^{-2} $arcsec^{-2}$)

$$\Sigma \equiv \frac{F}{\Omega} = \frac{L}{4\pi\, d_L^{\,2}} \frac{d_A^{\,2}}{A} = \frac{L}{4\pi\, A\,(1+z)^4}$$

› Decreases very rapidly with z because:

- expansion spreads out the photons
- decreases their energy
- decreases their arrival rate

› Makes observing extended sources at high redshift (i.e. galaxies) much harder than it would be in a static (non-expanding) Universe.

5.3 Distance vs. redshift

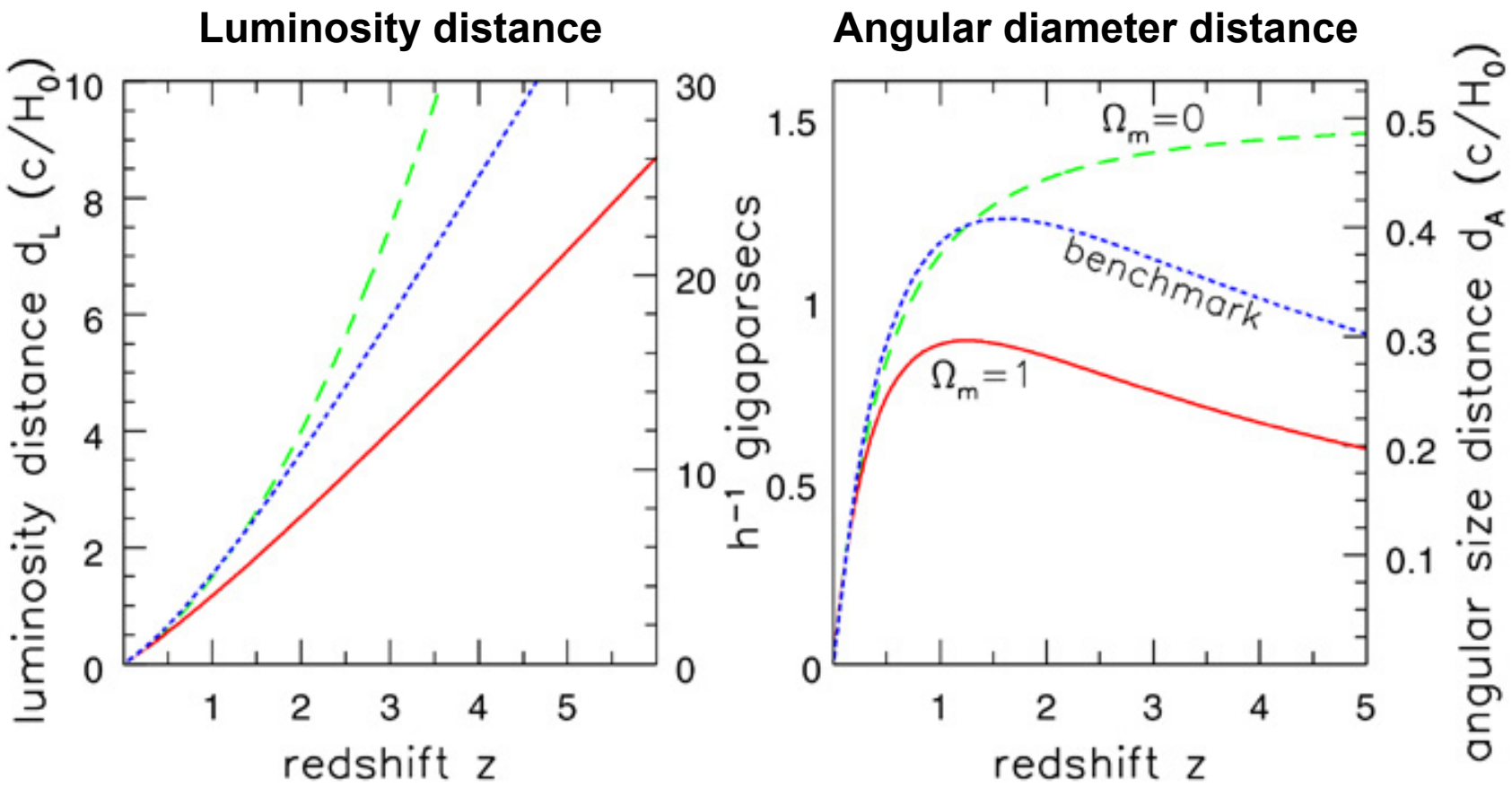


Fig 8.11 'Galaxies in the Universe' Sparke/Gallagher CUP 2007

Example flat Universes. Benchmark: Ω_m=0.3, Ω_Λ=0.7 (currently preferred model).

5.3 Distances summary

- Comoving distance (d_c)
  - Distance between objects with respect to the expanding metric. I.e. expansion removed.
  - Normalized to now (a_0) so is the proper (physical) distance we could (in principle) measure with a ruler now.
- Luminosity distance (d_L)
  - Distance the photon feels it has travelled.
  - For calculating luminosities.
- Angular diameter distance (d_A)
  - Proper (physical) distance at time of photon emission.
  - For calculating the sizes of bound structures.

$$d_c = (1+z)d_A = \frac{d_L}{(1+z)}$$

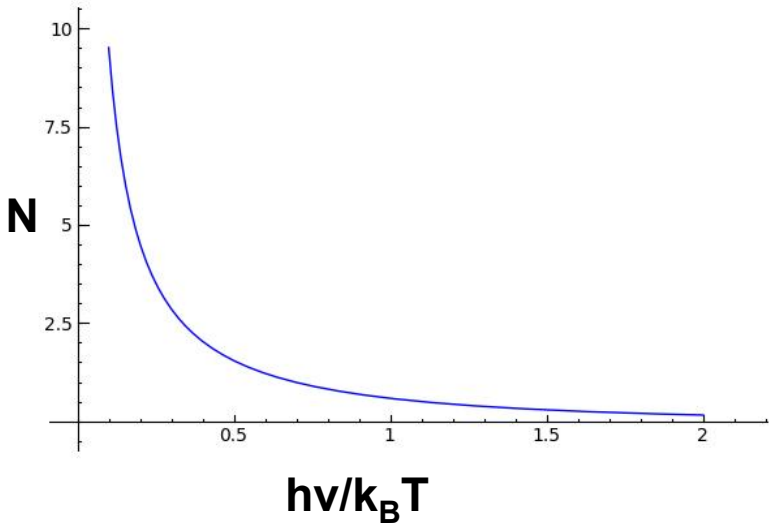
- Useful calculators:
  - http://www.astro.ucla.edu/~wright/CosmoCalc.html
  - http://icosmos.co.uk/ - includes graphing facility

5.4 Black-body distributions

- If particles are frequently interacting with each other, they are in thermal equilibrium: forwards and backwards interactions proceed at the same rate.
- In this case the particle numbers and energies do not change.
- Then the number of particles of a given energy only depends to temperature, T.
- For photons the number vs. energy(=hv) is given by a **Planck or Black-body** spectrum:

$$N = \frac{1}{\exp(h\nu / k_B T) - 1}$$

- Much easier to make photons at low energies.



5.4 Black-body distributions

› From the number of photons of a given energy it is possible to then derive the energy distribution (in this case the volume energy density):

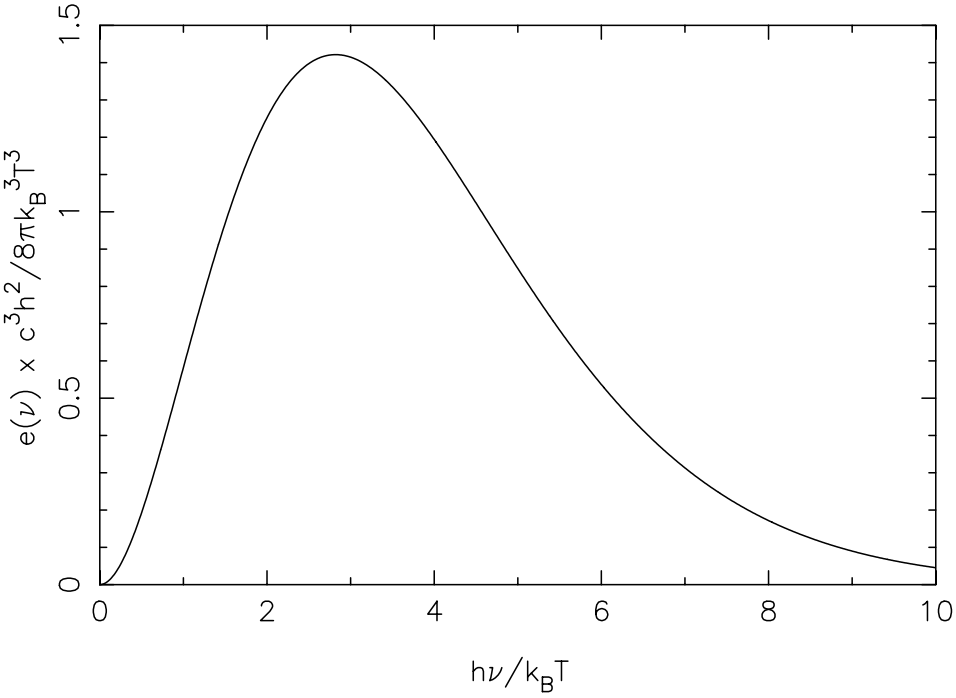
$$\varepsilon(\nu)d\nu = \frac{8\pi h}{c^3}\frac{\nu^3 d\nu}{\exp(h\nu/k_B T)-1}$$

Peak of distribution is at

$$E_{peak} = h\nu_{peak} \approx 2.8 k_B T.$$

Mean of distribution is

$$E_{mean} = h\nu_{mean} \approx 3 k_B T.$$



5.5 The cosmic microwave background

› Integrating the black-body function we can find the total energy density, which is

$$\varepsilon_{rad} = \alpha T^4$$

Where α is called the radiation constant :

$$\alpha = \frac{8\pi^5 k_B^4}{15h^3c^3} = 7.565 \times 10^{-16} Jm^{-3}K^{-4}$$

and

$$\rho_{rad}c^2 = \varepsilon_{rad} = \alpha T^4$$

Measured CMB temperature is

$$T_0 = 2.725 \pm 0.001\ K.$$

$$\text{So } \varepsilon_{rad}(t_0) = 4.17 \times 10^{-14} Jm^{-3} \Rightarrow \rho_{rad} = \frac{\varepsilon}{c^2} = 4.5 \times 10^{-31} kgm^{-3}$$

5.5 The cosmic microwave background

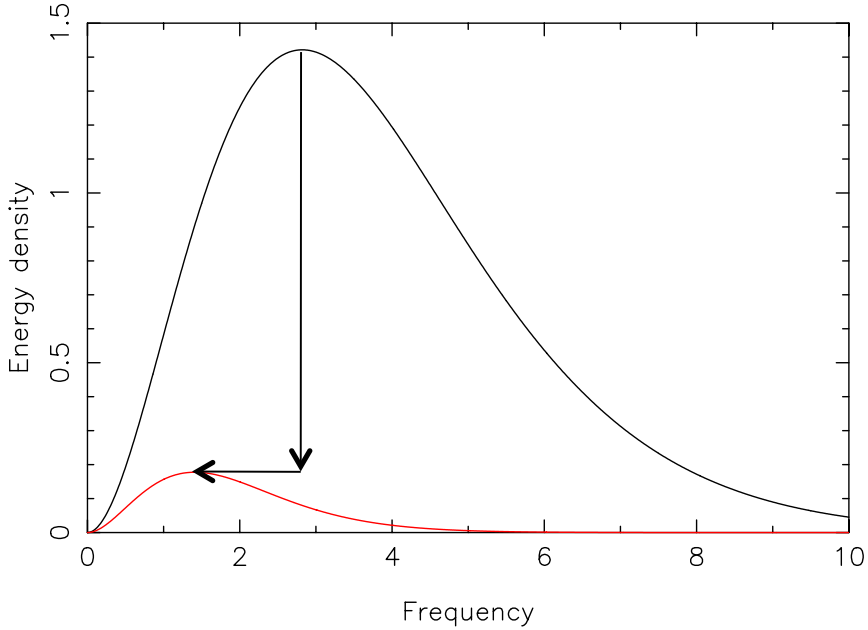
› We know that

$$\rho_{rad} c^2 = \varepsilon_{rad} = \alpha T^4$$

but recall that $\rho_{\rm rad}$ evolves as

$$\rho_{rad} \propto \frac{1}{a^4}$$

which implies that : $$T \propto \frac{1}{a}$$



› BB spectrum maintains its shape as T changes.

› Can maintain a thermal distribution, as long as some time in the past the system was in thermal equilibrium.

$$\varepsilon_\nu d\nu = \frac{8\pi h}{c^3} \frac{\nu^3}{\exp(h\nu / k_B T) - 1} d\nu$$

with expansion : $\mathrm{T} \downarrow \propto \nu \downarrow$

Shape dependent on exponential term only

i.e., $\varepsilon_{\nu_1} d\nu_1 = a^4 \varepsilon_{\nu_o} d\nu_o$

if a is the expansion factor between ν_1 and ν_0

5.5 The photon-to-baryon ratio

$$\text{Photon energy density} \approx 4.5 \times 10^{-31}\,\text{kgm}^{-3}$$

$$\text{Energy of mean photon} \approx 3k_B T = 0.00023\,\text{eV}$$

$$\text{Photon density} \approx 1.9 \times 10^{-27}\,\text{m}^{-3}$$

$$\text{Baryon mass density} \approx 2 \times 10^{-27}\,\text{kgm}^{-3}$$

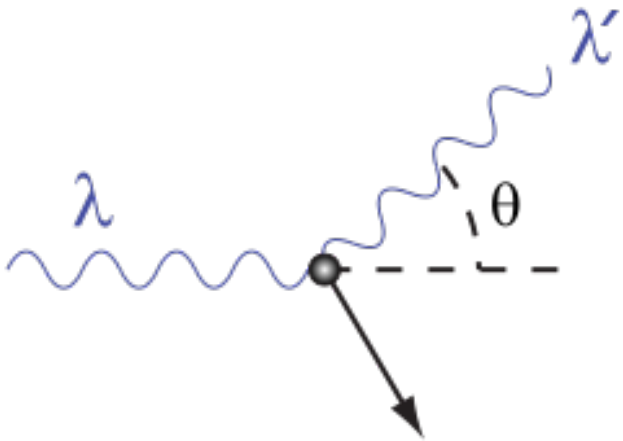
$$\text{Energy of mean baryon} \approx 939\,\text{MeV}$$

$$\text{Baryon density} \approx 2.1 \times 10^{-36}\,\text{m}^{-3}$$

$$\text{Photon} - \text{to} - \text{baryon ratio} \approx 10^{9}$$

5.5 Origin of the CMB

- As T~1/a, at earlier times, with smaller scale factor, the Universe would have been hotter – leads to the picture of a ***Hot Big Bang***.
- Currently the CMB photons do not interact significantly with matter.
- As temperature increases back in time eventually the radiation will be hot enough to ionize the neutral atoms and a plasma will be formed.
- In a plasma there is a large cross-section for interaction between the photons and electrons (i.e. Thompson scattering).
- Electrons and photons maintain thermal equilibrium.

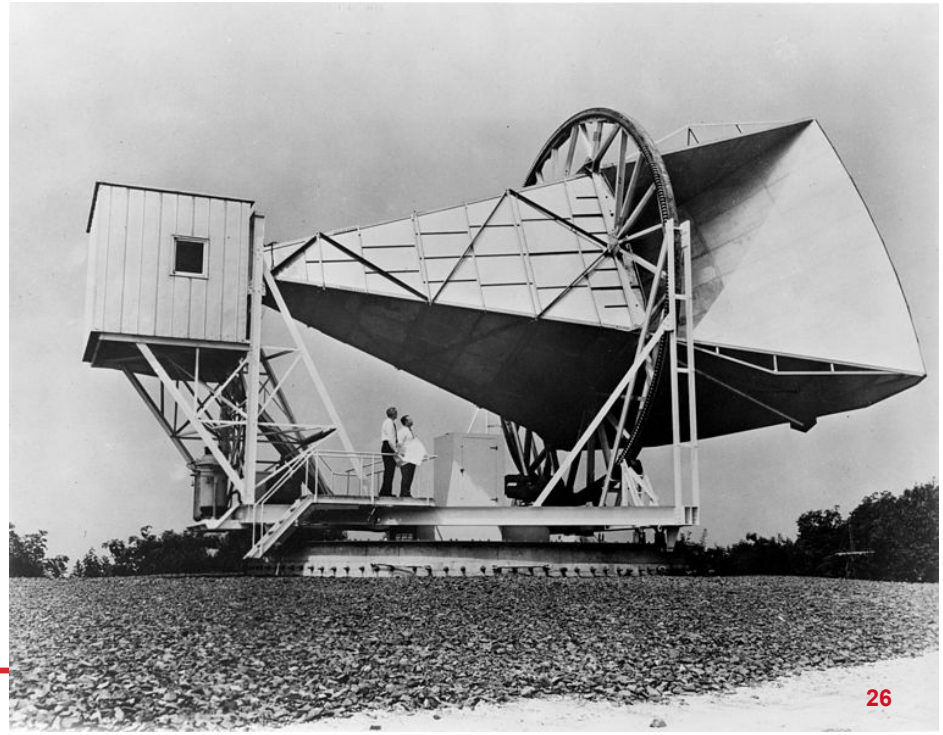


5.5 origin of CMB

- As the Universe cools, the electrons and protons will eventually be able to form stable atoms.
- The cross section for interactions between neutral atoms and photons is much lower.
- CMB photons free stream from this point of ***decoupling*** or ***"recombination"*** to us.
- When did decoupling occur?
- Need at least one photon per atom with energy higher than 13.6eV to maintain ionization.
- The BB temperature for this energy is $$T = \frac{13.6eV}{3k_B} \approx 50{,}000K$$
- But x10^9 more photons means that the tail of the BB distribution can maintain ionization. Detailed calculations give: $$T \approx 3000K$$

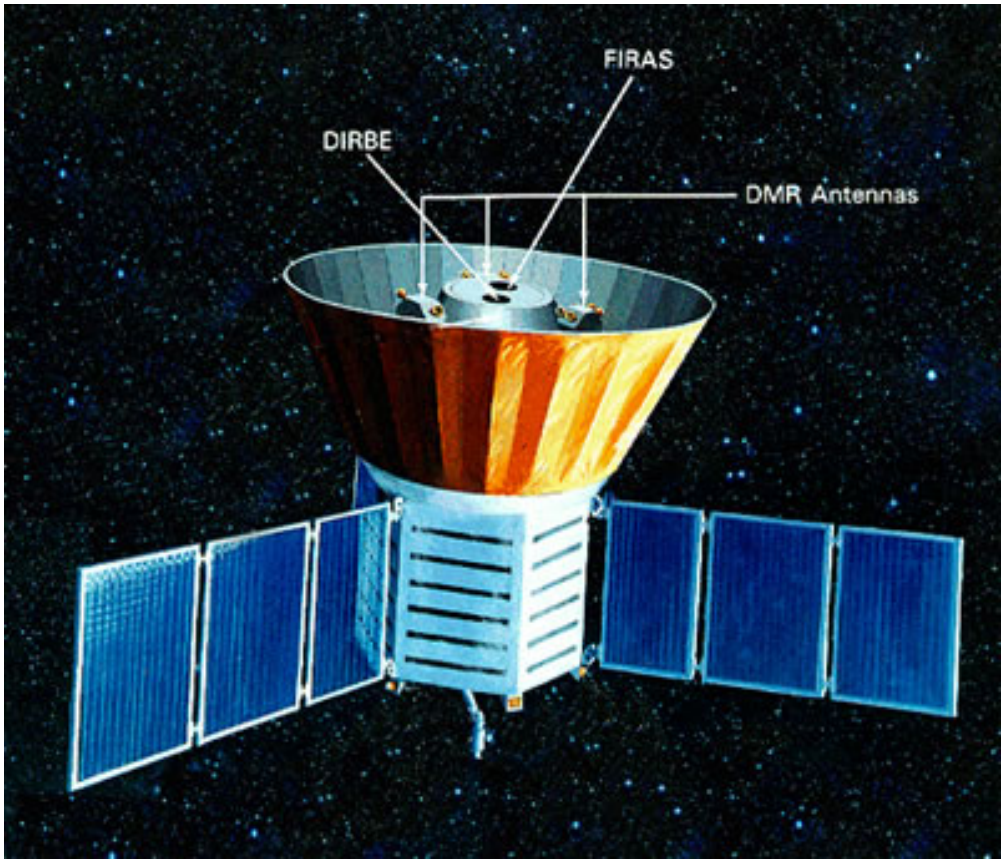
5.6 Observations of the CMB

- 1948 George Gamov predicted that if a Big Bang occurred a perfect black body spectrum should exist with T~5K.
- No attempt to look for this was made until 1964
  - Dicke radiometer at Princeton.
- In 1964 Penzias & Wilson discovered a background radiation at 3.5K
- Perfect BB spectrum (later)
- Nobel prize 1978.

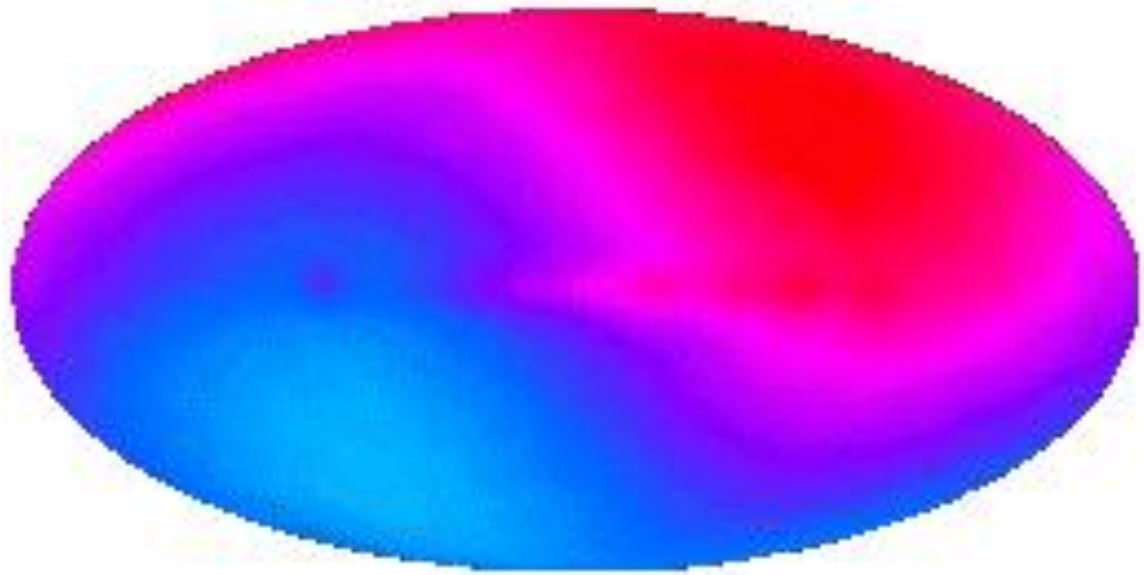


5.6 COBE satellite

- Launched 1989
- COsmic Background Explorer.
- Aimed to measure both the spectrum and anisotropies in the CMB
- 3 instruments:
  - DMR
  - FIRAS
  - DIRBE

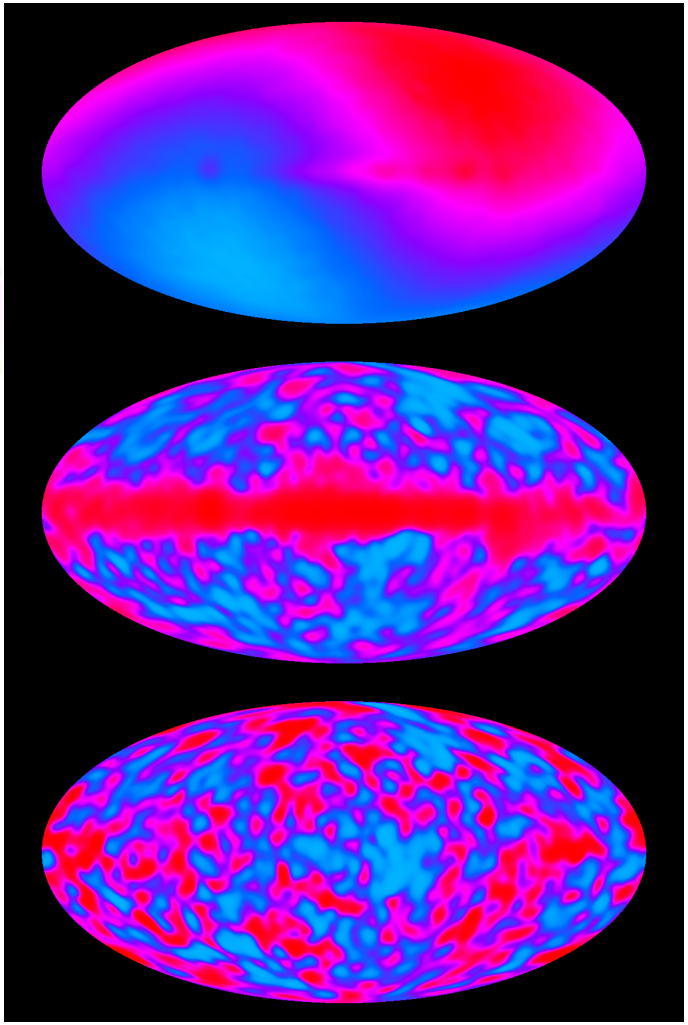


5.6 COBE satellite results



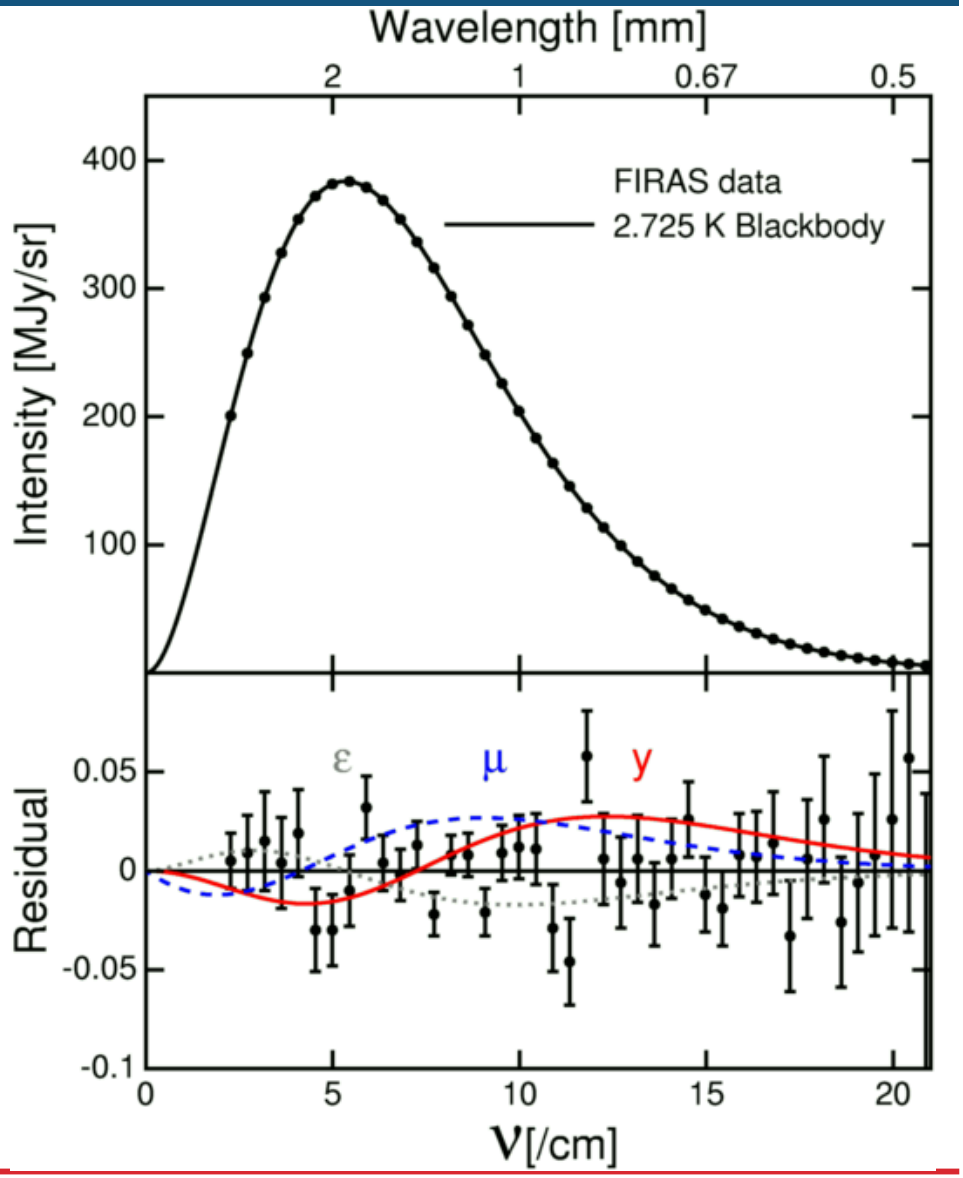
$$\frac{\Delta T}{T} \approx \frac{v}{c} \rightarrow v \approx 600 \text{ km s}^{-1}$$

Dipole shows the velocity of the Milky Way with respect to the CMB.



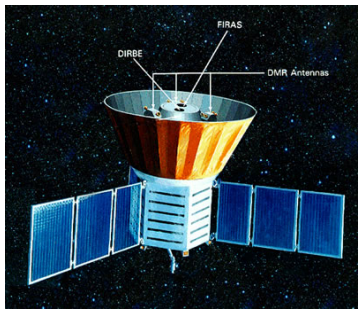
5.6 COBE satellite results

- Near perfect BB spectrum with peak at 2.725K.
- Nobel Prize awarded for spectrum and anisotropies in 2006.



5.6 CMB observations

COBE 1994



WMAP 2004

