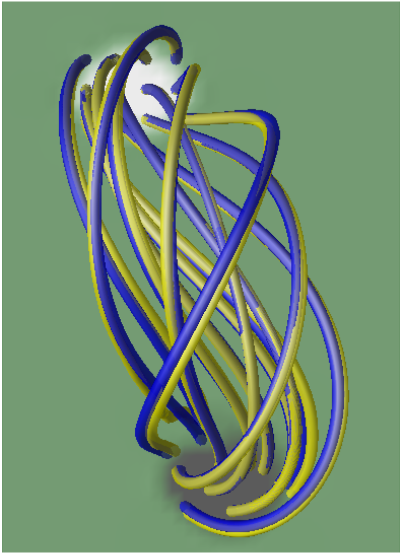
A fast current-field iteration approach to nonlinear force-free fields

M.S. Wheatland

School of Physics, University of Sydney

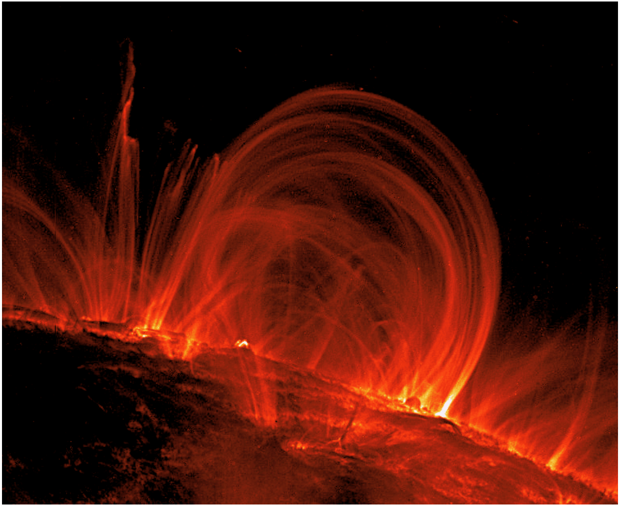
Berkeley SSL, 9 June 2006



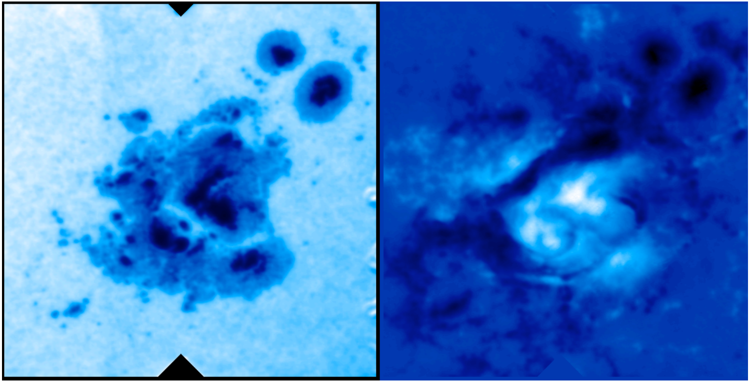
Overview

Solar coronal magnetic fields

- Magnetic fields around sunspots power solar flares



- Spectro-polarimetric instruments provide vector magnetic fields in the low atmosphere



Data: T. Metcalf & K.D. Leka

- Boundary value problem to determine **B** in the corona?

Force-free fields

- Force-free magnetic field **B**

$$(\nabla \times \mathbf{B}) \times \mathbf{B} = 0, \qquad \nabla \cdot \mathbf{B} = 0 \tag{1}$$

  - model for static fields in solar corona
  - current density $\mathbf{J} = \mu_0^{-1} \nabla \times \mathbf{B}$ parallel to **B**
- Alternative form:

$$\nabla \times \mathbf{B} = \alpha \mathbf{B} \tag{2}$$

$$\mathbf{B} \cdot \nabla \alpha = 0 \tag{3}$$

  - $\alpha = 0$, $\alpha = \mathrm{const}$: potential, linear force-free fields
  - $\alpha = \alpha(\mathbf{r})$: nonlinear force-free fields

- Variety of solution methods proposed
  - current-field iteration (Grad & Rubin, 1958)
  - magneto-frictional (Chodura & Schlüter, 1981)
  - optimization (Wheatland, Sturrock & Roumeliotis, 2000)
- NLFFF Workshop 2005 (Schrijver et al., 2006)
  - methods tested on Low & Lou (1990) cases
  - winner: optimization, as implemented by Wiegelmann (Wiegelmann, 2004)

Problem of slow force-free methods

- Order: time as a function of N for N^3 points
- Nonlinear force-free methods are slow
  - optimization: $O(N^5)$ [Inhester & Wiegelmann, 2006: $O(N^4)$]
  - magnetofrictional: $O(N^5)$ (Valori, Kliem & Keppens, 2006)
  - current-field iteration: $O(N^6)$ (Amari et al., 1999; Wheatland, 2004)
- New instruments are high resolution, $N \approx$ 1k–2k
  - SOLIS/VSM: $N \approx 2k$
  - SDO/HMI: $N = 4k$
  - Solar-B/SOT: $N =$ 1k–2k
- Ideally want a method approaching $O(N^3)$

Current-field iteration

- Grad & Rubin (1958) approach:

$$\nabla \times \mathbf{B}^{k+1} = \alpha^k \mathbf{B}^k \tag{4}$$

$$\mathbf{B}^{k+1} \cdot \nabla \alpha^{k+1} = 0 \tag{5}$$

with

$$\hat{\mathbf{z}} \cdot \mathbf{B}^k \Big|_{z=0} = \hat{\mathbf{z}} \cdot \mathbf{B}^{\mathrm{obs}} \Big|_{z=0}, \tag{6}$$

$$\alpha^k \Big|_{z=0, B_z>0} = \alpha^{\mathrm{obs}} \Big|_{z=0, B_z>0} \tag{7}$$

- Various implementations (e.g. Sakurai 1981; Amari et al. 1999; Wheatland 2004; Inhester & Wiegelmann 2006)
  - some specific implementations $O(N^6)$

New implementation

- Separate into potential, non-potential parts:

$$\mathbf{B}^{k+1} = \mathbf{B}_0 + \mathbf{B}_c^{k+1} \tag{8}$$

where

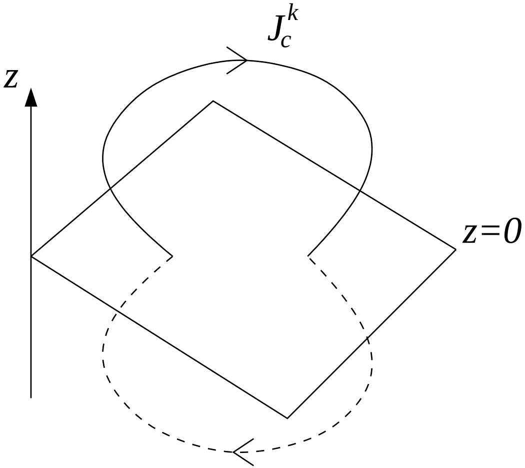
$$\nabla \times \mathbf{B}_0 = 0 \quad \text{and} \quad \hat{\mathbf{z}} \cdot \mathbf{B}_0\big|_{z=0} = \hat{\mathbf{z}} \cdot \mathbf{B}^{\mathrm{obs}}\Big|_{z=0} \tag{9}$$

- $\mathbf{B}_c^{k+1}$ may be constructed by solving

$$\nabla \times \mathbf{B}_c^{k+1} = \mu_0 \mathbf{J}_c^k \tag{10}$$

where (Sakurai 1981)

$$\mathbf{J}_c^k = \begin{cases} \alpha^k \mathbf{B}^k / \mu_0 & (z \geq 0) \\ \left[-J_{cx}^k(x,y,-z), -J_{cy}^k(x,y,-z), J_{cz}^k(x,y,-z)\right] & (z < 0) \end{cases}$$



- Construction ensures

$$\hat{\mathbf{z}} \cdot \mathbf{B}_c^{k+1}\Big|_{z=0} = 0 \qquad (11)$$

as required by Eqs. (6) and (9)

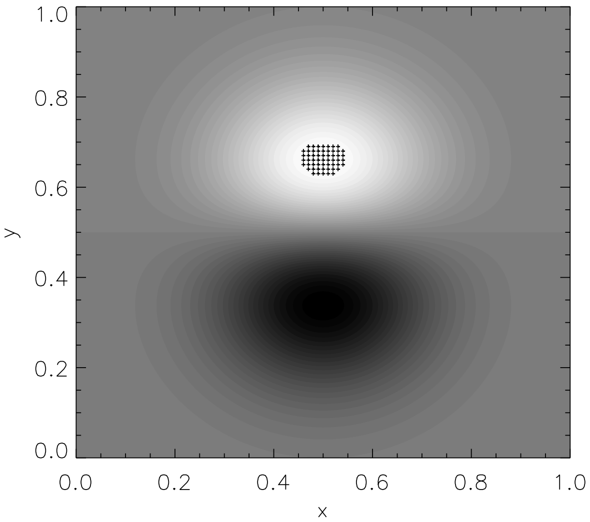
- Write $\mathbf{B}_c^{k+1} = \nabla \times \mathbf{A}_c^{k+1}$, solve

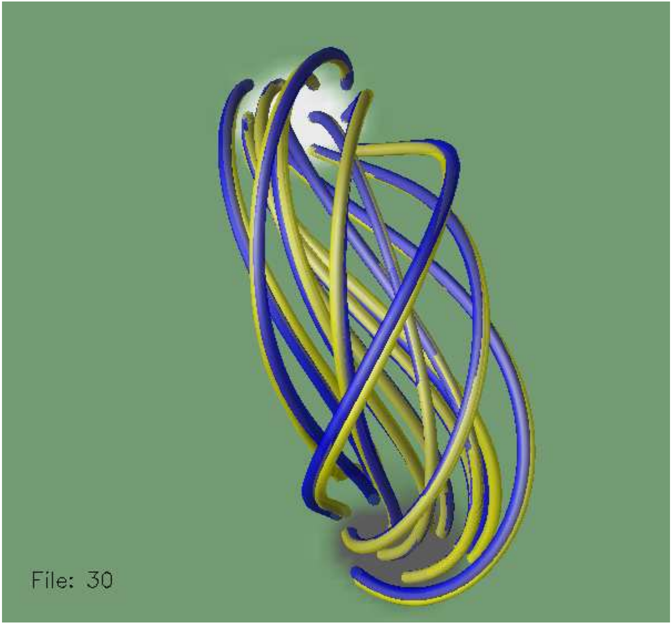
$$\nabla^2 \mathbf{A}_c^{k+1} = -\mu_0 \mathbf{J}_c^k$$

  $(\nabla \cdot \mathbf{A}_c^{k+1} = 0)$ via 3-D FFTs with no explicit BCs
  - periodic — necessary to pad with zeros
  - $O(N^3 \log N^3)$ operations
- $\mathbf{B}^{k+1} \cdot \nabla \alpha^{k+1} = 0$ solved by field line tracing
  - for each point $\mathbf{r}$, trace field line in both directions
  - if field line leaves box via sides or top, $\alpha^{k+1}(\mathbf{r}) = 0$
  - otherwise assign $\alpha^{k+1}(\mathbf{r})$ based on bilinear interpolation at $z = 0, B_z > 0$
  - $O(N^4)$ operations
- $\mathbf{B}_0$ is the starting field for the iteration
- Code is Fortran 90 using OpenMP, all arrays dynamic

Bipolar test case

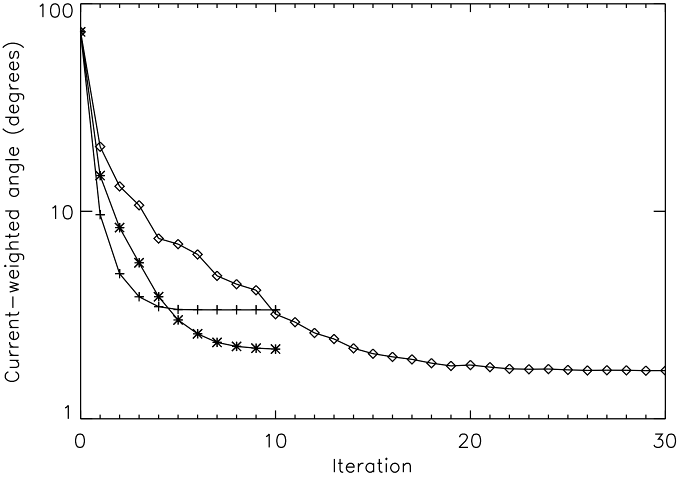
- Example used in Wheatland (2004)




File: 30

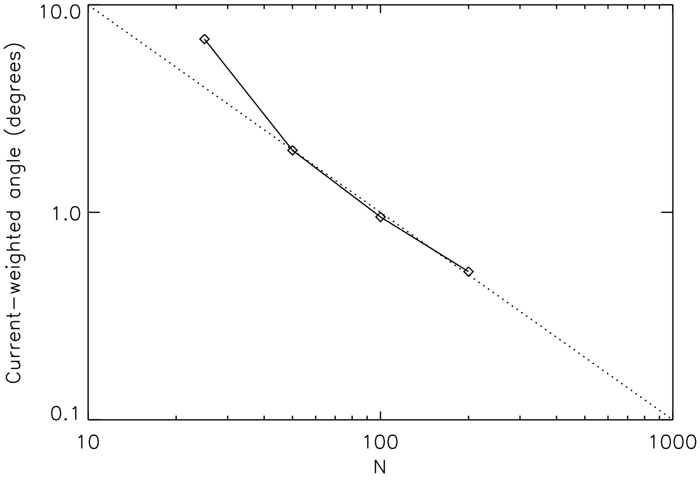
- Mean current-weighted angle between **J** and **B**:

$$\sin\theta_J = \sum_i |\mathbf{J}_i| \sin\theta_i \Big/ \sum_i |\mathbf{J}_i| \qquad (12)$$

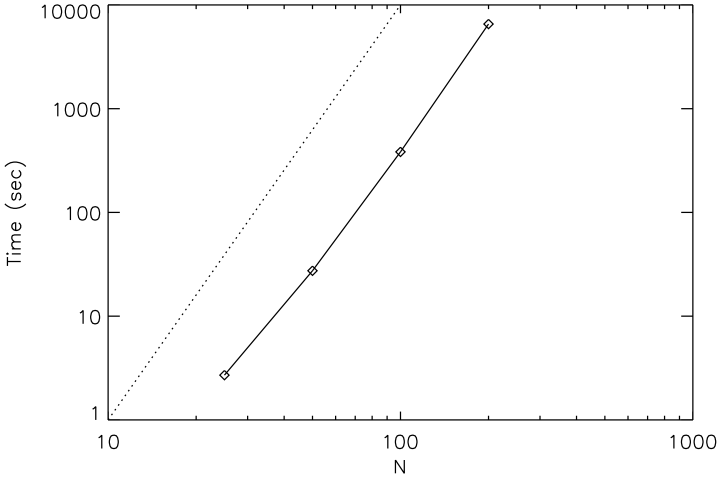


Low & Lou (1990) test case

- Test case 1 from NLFFF Workshop 2005 (Schrijver et al., 2006)

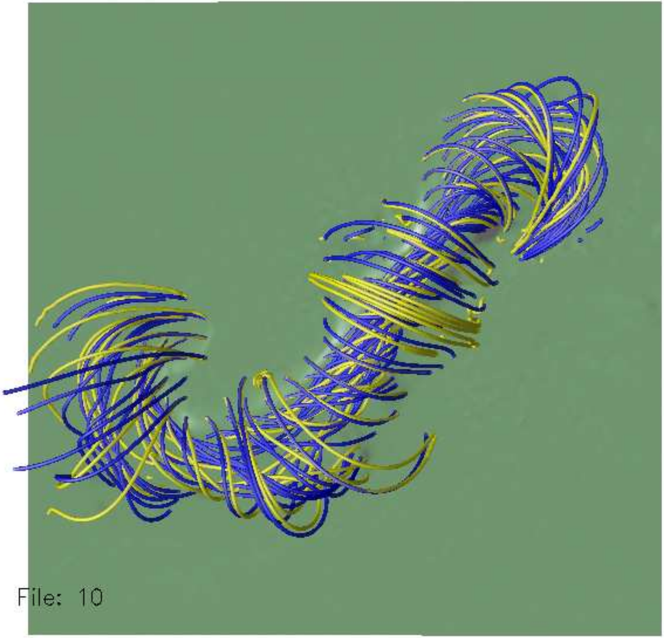


- Test of order of the method



NLFFF Workshop 2006 test case

- Test case supplied by Aad van Ballegooijen
  - solar-like test case
  - Aad started with MDI data, ‘inserted’ a flux rope, performed magnetofrictional relaxation
  - boundary data given to workshop participants
- Boundary data for calculation: 232×232
  - calculation on $232 \times 232 \times 212$ grid
- 10 iterations performed
- Time: 2.75 hours on 2×Opteron 270 (9.1 CPU hours)
- Result not a true fixed point of iteration
  - minimum angle between **J** and **B** after 7 iterations
  - different results if $\alpha|_{z=0}$ is increased at each iteration to match $\alpha^{\mathrm{obs}}|_{z=0}$ at final iteration
- After 10 iterations, current-weighted angle $\approx 10.5^\circ$


File: 10

Summary

- Nonlinear force-free fields model coronal fields
- Existing nonlinear methods are slow
  - inadequate for application to new datasets
- Fast current-field iteration method developed
  - FFT solution of Ampere's law using current density constructed to impose boundary conditions
  - field line tracing to solve α-updating step
- Method is $O(N^4)$, accurate, simple
  - performs well on bipole, Low & Lou (1990) tests
  - may be further optimized
- Encouraging results for Aad's test case
  - flux-rope like structure recovered