Nonlinear force-free magnetic fields

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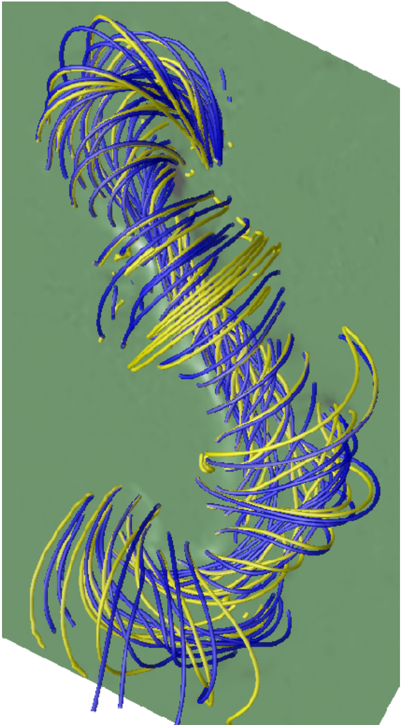
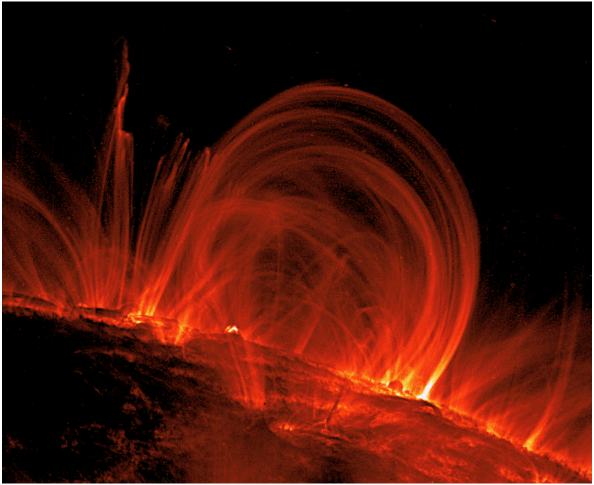


Overview

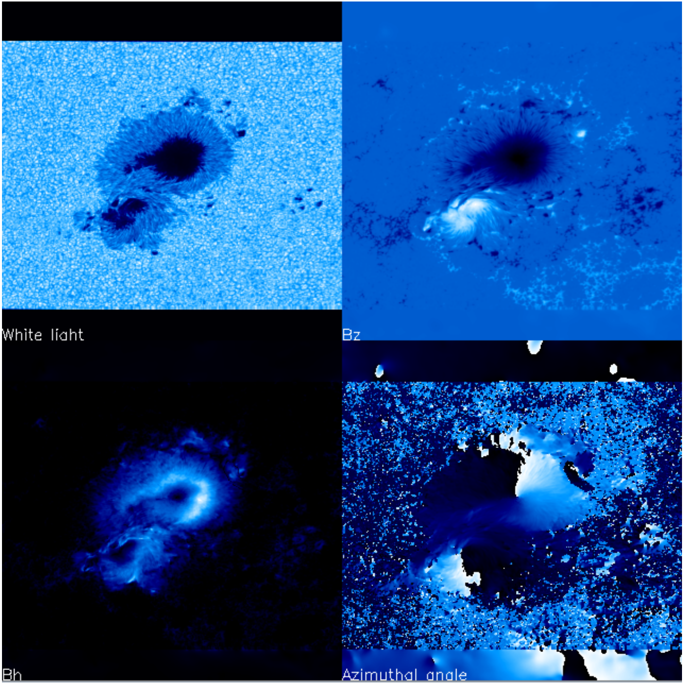
Solar coronal magnetic fields

- Magnetic fields around sunspots power flares, CMEs
- Space weather effects of large flares motivate flare prediction



Spectro-polarimetric data

- Vector magnetic fields in the low atmosphere may be inferred
- Problems:
  - instrumental uncertainties
  - validity/reliability of the inversion
  - 180 degree ambiguity in transverse field
- New generation of high resolution instruments
  - SOLIS/VSM (now): detector size $N \approx 2k$, $1''$, full disk
  - Hinode (now): $N = 1$k–2k, $0.16''$, small FOV
  - SDO/HMI (2008): $N = 4k$, $1''$, full disk
- Measurements may be used to:
  - investigate topology
  - determine free energy



Data: T.R. Metcalf

Nonlinear force-free magnetic fields

- Force-free magnetic field $\mathbf{B}$

$$(\nabla \times \mathbf{B}) \times \mathbf{B} = 0, \qquad \nabla \cdot \mathbf{B} = 0 \tag{1}$$

  - model for static fields in solar corona
  - current density $\mathbf{J} = \mu_0^{-1} \nabla \times \mathbf{B}$ parallel to $\mathbf{B}$
- Alternative form:

$$\nabla \times \mathbf{B} = \alpha \mathbf{B} \tag{2}$$

$$\mathbf{B} \cdot \nabla \alpha = 0 \tag{3}$$

  - $\alpha = 0$, $\alpha = \mathrm{const}$: potential, linear force-free fields
  - $\alpha = \alpha(\mathbf{r})$: nonlinear force-free fields
  - nonlinear case appropriate for solar modelling

- Can we solve the force-free equations using BCs from spectro-polarimetric instruments?
  - reconstruct coronal field from lower boundary values
- Photospheric field is not force-free
  - field force-free at a height $\approx 500\,\mathrm{km}$ (Metcalf et al., 1995)
- Variety of methods of solution of nonlinear force-free equations investigated
  - current-field iteration (Grad & Rubin 1958)
  - magneto-frictional (Chodura & Schlüter 1981)
  - optimization (Wheatland, Sturrock & Roumeliotis, 2000)
- NLFFF workshops
  - 2005: Low & Lou 1990 test cases (Schrijver et al., 2006)
  - 2006: solar-like test case
  - 2007: Hinode data

Force-free methods are slow

- Order: time as a function of N for N^3 points

Method	Order
Optimization	$O(N^5)$
Magnetofrictional	$O(N^5)$
Current-field iteration	$O(N^6)$

(Schrijver et al., 2006)

- New instruments are high resolution, $N \approx$ 1k–2k
  - calculations for this size unfeasible?
- Recent $O(N^4)$ methods
  - optimization (Inhester & Weigelmann 2006)
  - current-field iteration (Wheatland 2006)

Current-field iteration

- General approach (Grad & Rubin 1958):

$$\nabla \times \mathbf{B}^{k+1} = \alpha^k \mathbf{B}^k \tag{4}$$

$$\mathbf{B}^{k+1} \cdot \nabla \alpha^{k+1} = 0 \tag{5}$$

with

$$\hat{\mathbf{z}} \cdot \mathbf{B}^{k+1}\Big|_{z=0} = \hat{\mathbf{z}} \cdot \mathbf{B}^{\mathrm{obs}}\Big|_{z=0}, \tag{6}$$

$$\alpha^{k+1}\Big|_{z=0,B_z>0} = \alpha^{\mathrm{obs}}\Big|_{z=0,B_z>0} \tag{7}$$

and $\mathbf{B}^k = \mathbf{B}_0$, a potential field

- Various implementations (e.g. Sakurai 1981; Amari et al., 1999; Wheatland 2004, 2006; Inhester & Wiegelmann 2006)
  - specific implementations $O(N^4) - O(N^6)$

Fast current-field iteration

1. Field updating

- Separate into potential, non-potential parts:

$$\mathbf{B}^{k+1} = \mathbf{B}_0 + \mathbf{B}_{\mathrm{c}}^{k+1} \tag{8}$$

where

$$\nabla \times \mathbf{B}_0 = 0 \quad \text{and} \quad \hat{\mathbf{z}} \cdot \mathbf{B}_0\big|_{z=0} = \hat{\mathbf{z}} \cdot \mathbf{B}^{\mathrm{obs}}\Big|_{z=0} \tag{9}$$

so

$$\hat{\mathbf{z}} \cdot \mathbf{B}_{\mathrm{c}}^{k+1}\Big|_{z=0} = 0 \tag{10}$$

- $\mathbf{B}_c^{k+1}$ may be constructed by solving

$$\nabla \times \mathbf{B}_c^{k+1} = \mu_0 \mathbf{J}_{\mathrm{c}}^k \tag{11}$$

with $\mathbf{J}_{\mathrm{c}}^k = \alpha^k \mathbf{B}^k$, subject to (10)

- Writing $\mathbf{B}_{\mathrm{c}}^{k+1} = \nabla \times \mathbf{A}_{\mathrm{c}}^{k+1}$ with $\nabla \cdot \mathbf{A}_{\mathrm{c}}^{k+1} = 0$, solve

$$\nabla^2 \mathbf{A}_{\mathrm{c}}^{k+1} = -\mu_0 \mathbf{J}_{\mathrm{c}}^{k} \tag{12}$$

  subject to

$$A_{\mathrm{c}x}^{k+1}\Big|_{z=0} = A_{\mathrm{c}y}^{k+1}\Big|_{z=0} = 0, \quad \frac{\partial A_{\mathrm{c}z}^{k+1}}{\partial z}\bigg|_{z=0} = 0 \tag{13}$$

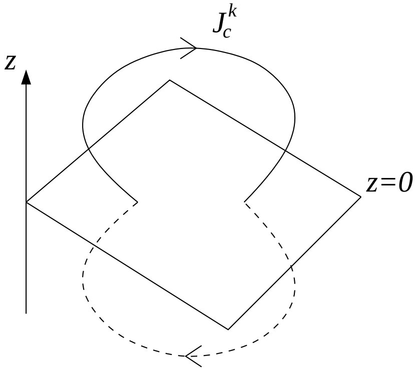
  and $\mathbf{A}_{\mathrm{c}}^{k+1} \to 0$ as $z \to \infty$

- Fourier transform in x and y to give

$$\frac{d^2 \widetilde{\mathbf{A}}_{\mathrm{c}}^{k+1}}{dz^2} - \kappa^2 \widetilde{\mathbf{A}}_{\mathrm{c}}^{k+1} = -\mu_0 \widetilde{\mathbf{J}}_{\mathrm{c}}^{k} \tag{14}$$

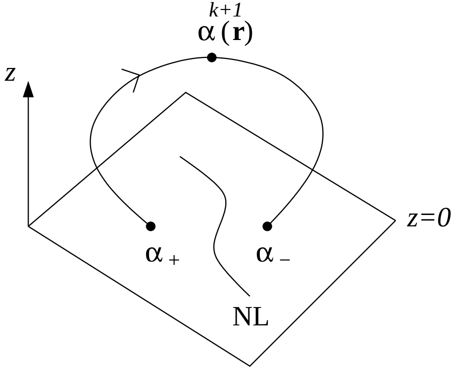
  with $\kappa^2 = 4\pi^2(u^2 + v^2)$, u and v are wave numbers
  - solve analytically with BCs to give $\widetilde{\mathbf{A}}_{\mathrm{c}}^{k+1}(u, v, z)$

- Using FT of $\mathbf{B}_c^{k+1} = \nabla \times \mathbf{A}_c^{k+1}$, obtain analytic expressions for $\widetilde{\mathbf{B}}_c^{k+1}$ in terms of $\widetilde{\mathbf{J}}_c^k$
- Procedure: FFT of $\alpha^k \mathbf{B}_c^k$ gives $\widetilde{\mathbf{J}}_c^k$
  - apply analytic solutions above to construct $\widetilde{\mathbf{B}}_c^{k+1}$
  - inverse FFT gives $\mathbf{B}_c^{k+1}$
- Solutions periodic in x and y: can pad with zeros
- $O(N^4)$ operations

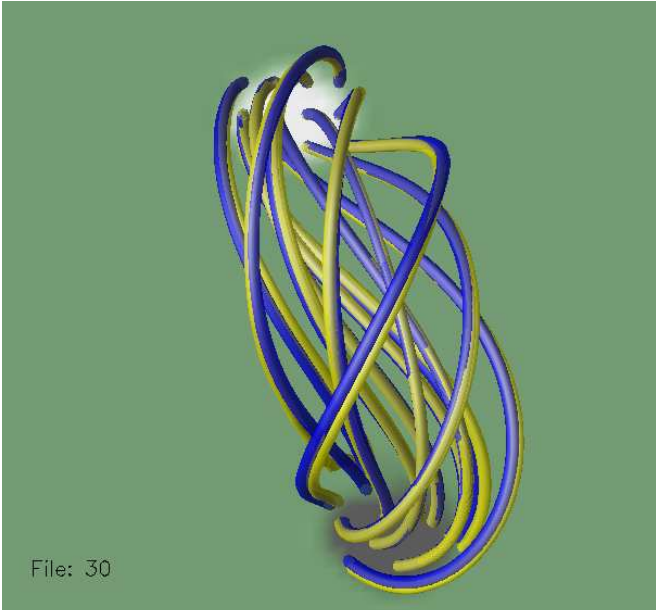


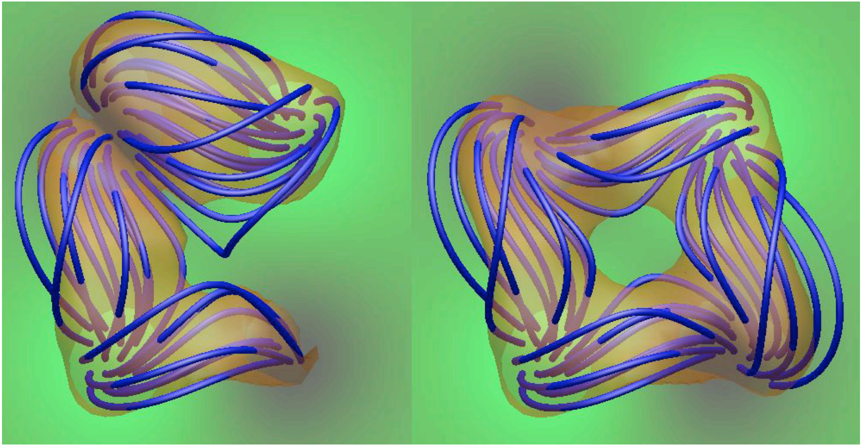
2. Current updating

- $\mathbf{B}^{k+1} \cdot \nabla\alpha^{k+1} = 0$ solved by field line tracing
  - for each point $\mathbf{r}$, trace field line in both directions
  - if field line leaves box via sides or top, $\alpha^{k+1}(\mathbf{r}) = 0$
  - if field line connects to $z = 0$ at both ends, $\alpha^{k+1}(\mathbf{r}) = \alpha_+$ (value at positive footpoint)
  - can also set $\alpha^{k+1}(\mathbf{r}) = \alpha_-$, or $\alpha^{k+1}(\mathbf{r}) = \frac{1}{2}(\alpha_+ + \alpha_-)$
  - $O(N^4)$ operations



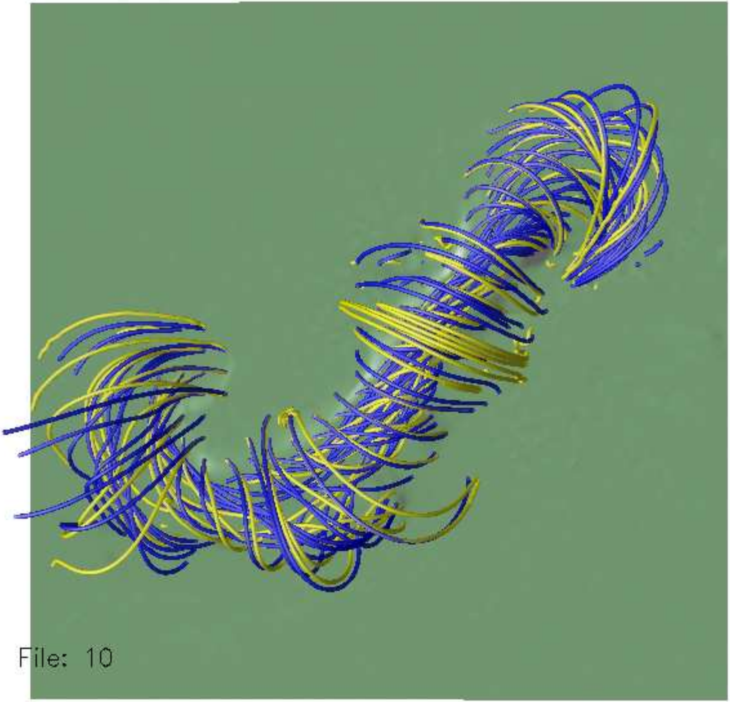
Examples





NLFFF workshop 2006

- Solar-like test case constructed by Aad van Ballegoojin
  - Aad started with MDI data, ‘inserted’ a flux rope, performed magnetofrictional relaxation
- Boundary data B_x, B_y, B_z given to workshop participants
- ‘Photospheric’ (forced) and ‘chromospheric’ (less forced) BCs provided; also preprocessed BCs
  - α obtained via $\alpha = B_z^{-1} (\partial B_y/\partial x - \partial B_x/\partial y)$
- Current-field iteration: approximate reconstruction achieved
  - 10 iterations on a $320 \times 320 \times 256$ grid
  - results not fixed points of iteration
  - current-weighted average angle between **J** and **B** $\approx 10\,\mathrm{deg}$
- $E/E_{\mathrm{pot}} < 1$ for magnetofrictional, optimization methods!
- Paper submitted (Metcalf et al. 2007)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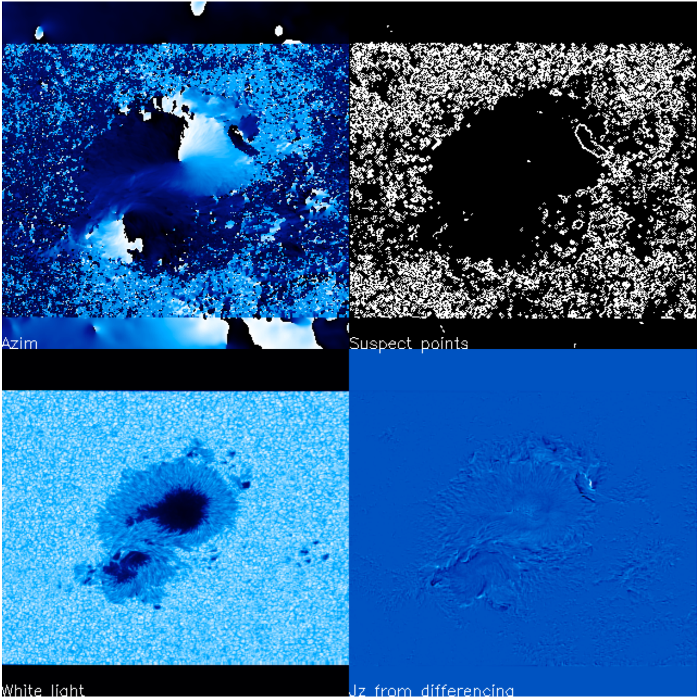
Model	C_{vec}	C_{CS}	$1-E_n$	$1-E_m$	CWsin	E/Epot	1-FLD (area/flux)
Chromospheric Boundary							
Reference Model	1.00	1.00	1.00	1.00	0.10	1.34	1.00/1.00
Wiegelmann	1.00	0.99	0.89	0.73	0.11	1.34	0.85/0.76
Wheatland	0.95	0.98	0.79	0.70	0.15	1.21	0.51/0.63
Valori	0.98	0.98	0.84	0.71	0.16	1.26	0.56/0.58
McTiernan	0.97	0.97	0.80	0.69	0.22	1.15	0.76/0.70
Potential Solution	0.85	0.96	0.69	0.67	(undef.)	1.00	0.47/0.57
Photospheric Boundary							
Reference Model	1.00	1.00	1.00	1.00	0.10	1.52	1.00/1.00
Wiegelmann	0.93	0.96	0.62	0.61	0.38	0.76	0.56/0.41
Wheatland	0.86	0.95	0.66	0.63	0.20	1.01	0.33/0.48
Valori	0.80	0.83	0.36	-0.08	0.46	0.46	0.35/0.29
McTiernan	0.91	0.96	0.63	0.63	0.47	0.75	0.56/0.46
Potential Solution	0.85	0.95	0.66	0.64	(undef.)	1.00	0.41/0.53
Photospheric Preprocessed Boundary							
Reference Model	1.00	1.00	1.00	1.00	0.10	1.52	1.00/1.00
Wiegelmann							
Smoothed	0.98	0.97	0.77	0.65	0.26	1.18	0.24/0.48
Unsmoothed	0.95	0.96	0.69	0.63	0.30	0.97	0.60/0.51
Wheatland							
Smoothed	0.88	0.96	0.69	0.65	0.11	1.03	0.22/0.47
Unsmoothed	0.82	0.93	0.59	0.56	0.38	1.08	0.13/0.26

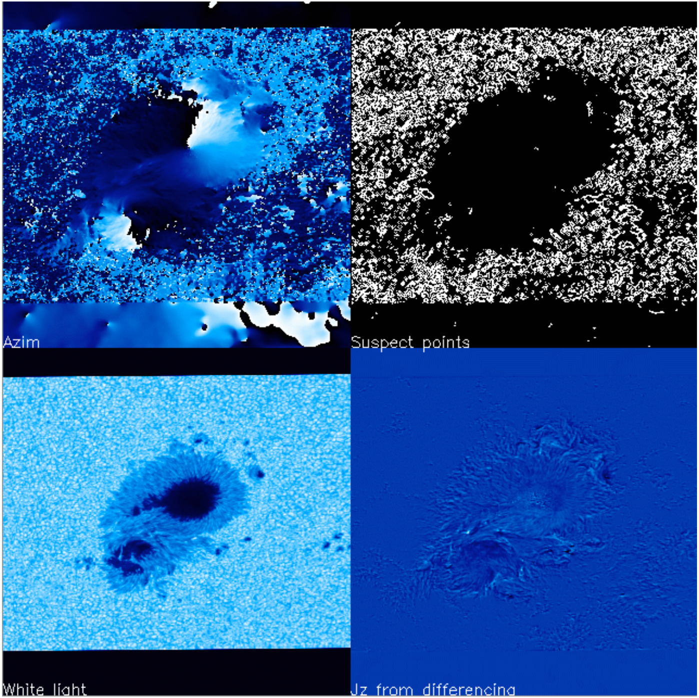
NLFFF workshop 2007

- Hinode data
  - rebinned to $0.63''$
  - embedded in MDI data to increase FOV (final size 320×320)
- BCs obtained via $\alpha = B_z^{-1} (\partial B_y / \partial x - \partial B_x / \partial y)$
- Problem of 180-degree ambiguity in azimuthal angle ϕ
  - Metcalf (1994) procedure was used
  - imperfect: spurious currents result
- Current-field iteration tends to be unstable
  - α values 'censored' on basis of B_z and $\Delta\phi \approx 180$ deg
  - under-relaxation:

$$\mathbf{B}^{k+1} = (1 - f)\mathbf{B}^k + f \left(\mathbf{B}_{\mathrm{c}}^{k+1} + \mathbf{B}_0 \right) \tag{15}$$

with $f < 1$


Azim
Suspect points
White light
Jz from differencing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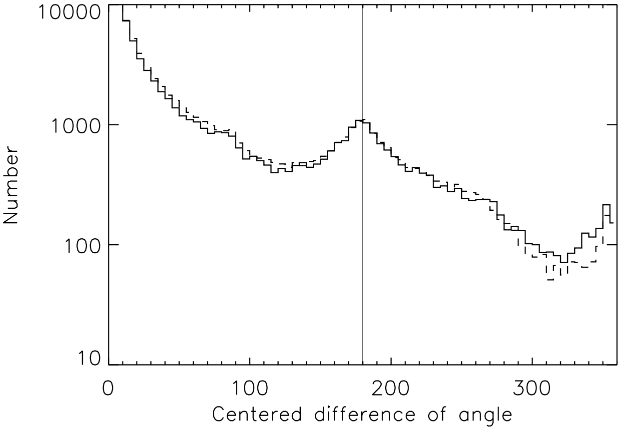
10000
1000
100
10
Number
0
100
200
300
Centered difference of angle

Summary

- Nonlinear force-free fields provide a zeroth order model for coronal magnetic fields
  - boundary value problem of 'reconstructing' coronal field from **B** values in low atmosphere
- Nonlinear force-free fields difficult to calculate, methods slow
  - recently some $O(N^4)$ methods have been developed
- Current-field iteration method described
- NLFFF workshops 2006 and 2007 described