A self-consistent nonlinear force-free solution for a solar active region magnetic field

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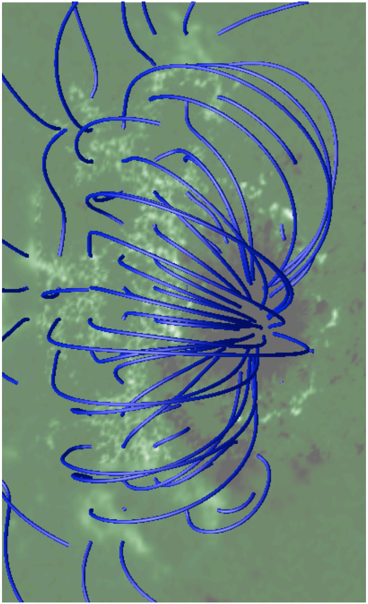


Overview

Background

- *Solar flares*
- *Vector magnetograms*
- *Nonlinear force-free model*
- *The problem – two solutions*

Method

- *Using both solutions*
- *Bayesian decision making*
- *Iteration*

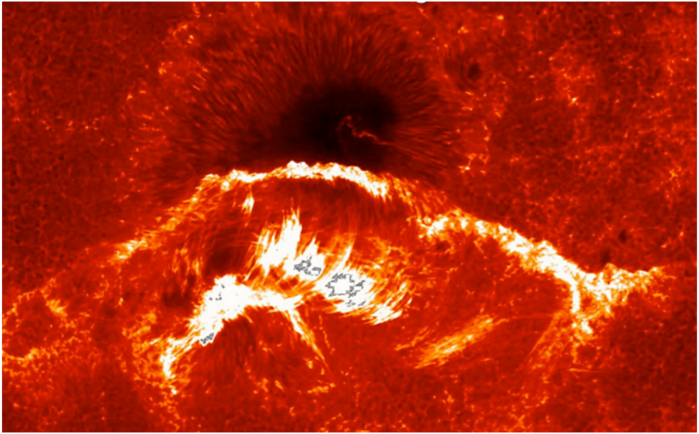
Application to Hinode/SOT data

- *AR 10953*

Summary

Background: Solar flares

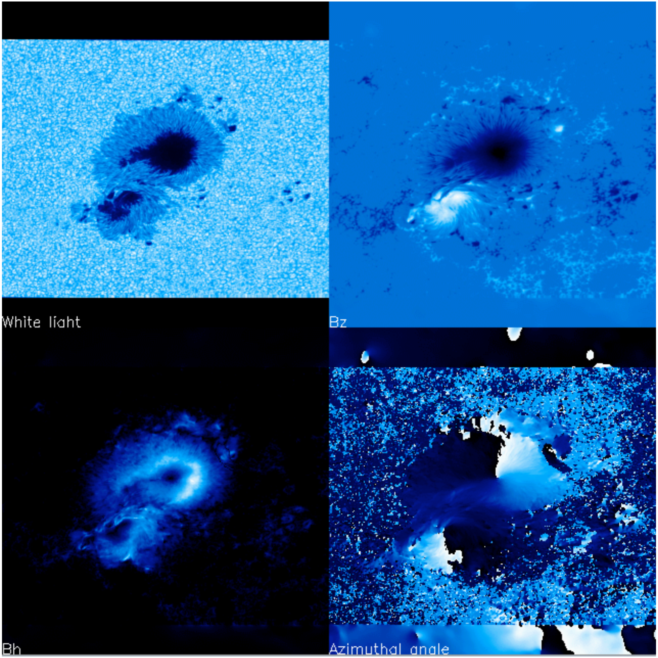
- Magnetic explosions in the Sun's corona
  - large flares influence local "space weather"
- Need to accurately model the coronal field



Data: Hinode/SOT

Background: Vector magnetograms

- Polarisation state of photospheric lines measured
- Vector magnetic field inferred ("inversion")
  - map of **B** at photosphere ("vector magnetogram")
- Problems:
  - instrumental uncertainties
  - validity/reliability of the inversion
  - 180 degree ambiguity in transverse field
- New generation of high resolution instruments
  - SOLIS/VSM: ground based, full disk
  - Hinode/SOT: satellite launched in 2006
  - SDO/HMI: to be launched in 2009
- In principle, boundary conditions for coronal field modelling



Data: Hinode/SOT (Schrijver et al. 2008)

Background: Nonlinear force-free magnetic fields

- Force-free field $\mathbf{B}$ satisfies

$$\begin{aligned} (\nabla \times \mathbf{B}) \times \mathbf{B} &= 0 \\ \nabla \cdot \mathbf{B} &= 0 \end{aligned} \tag{1}$$

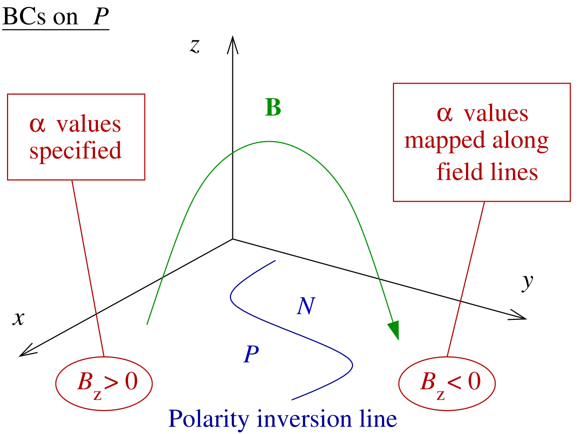
  - suitable model for the coronal magnetic field
  - current density $\mathbf{J} = \mu_0^{-1} \nabla \times \mathbf{B}$ is parallel to $\mathbf{B}$
  - coupled nonlinear PDEs

- Alternative form:

$$\begin{aligned} \nabla \times \mathbf{B} &= \alpha \mathbf{B} \\ \mathbf{B} \cdot \nabla \alpha &= 0 \end{aligned} \tag{2}$$

  - force-free parameter α is constant along field lines

- Boundary conditions (Grad & Rubin 1958):
  - B_n in boundary
  - α in boundary over region where $B_n > 0$ *or* where $B_n < 0$
    - over "one polarity"
    - we label the polarities P and N respectively

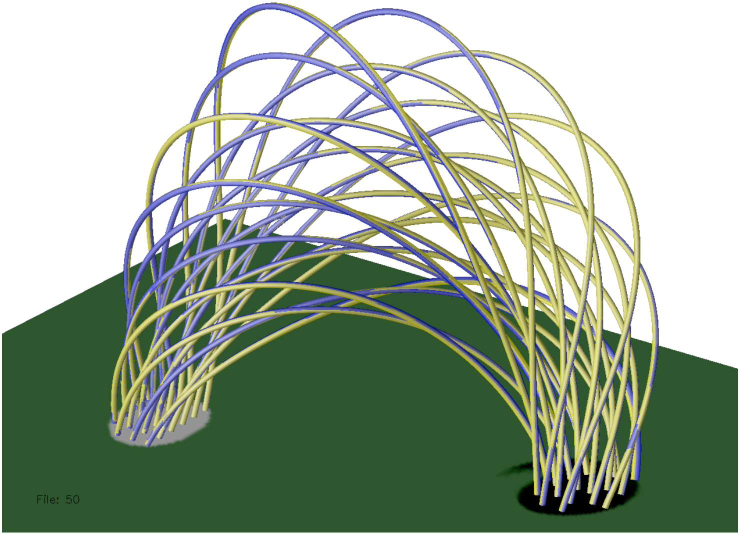


- Note that $\mathbf{J} = \alpha \mathbf{B}/\mu_0$
  - alternatively, BCs for $J_z = \alpha B_z/\mu_0$ over P or over N
- Nonlinear force-free BVPs are hard to solve
- Variety of iterative numerical methods (Wiegelmann 2008)
  - demonstrated to work on test cases (Schrijver et al. 2006)
  - not all methods use correct BCs
- Current-field iteration (Grad & Rubin 1958)
  - "Picard" iteration: at iteration $k+1$, solve

$$\begin{aligned} \nabla \times \mathbf{B}^{k+1} &= \alpha^k \mathbf{B}^k \\ \mathbf{B}^{k+1} \cdot \nabla \alpha^{k+1} &= 0 \end{aligned} \tag{3}$$

- Fast current-field iteration (Wheatland 2007)
  - order N^4 (grid with N^3 points)
  - parallel code (OpenMP)

- Bipole test case
  - **B**-lines blue, **J**-lines yellow



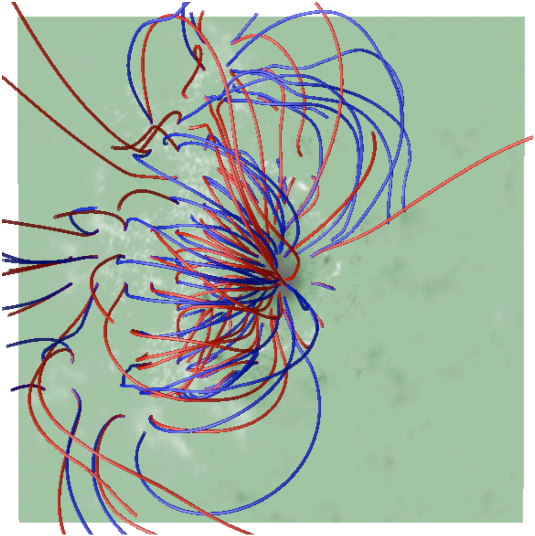
Background: The problem – two solutions

- Force-free modelling from solar data
  - assume locally planar photosphere ($z = 0$ plane)
  - $B_z(x, y, 0)$ from vector magnetogram
  - $\alpha(x, y, 0)$ from magnetogram values via

$$\alpha(x, y, 0) = \frac{1}{B_z} \left(\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y} \right) \bigg|_{z=0} \tag{4}$$

  for points with $B_z(x, y, 0) > 0$ (P) *or* $B_z(x, y, 0) < 0$ (N)
- Workshops on application to Hinode/SOT data
  - different force-free methods produce different solutions (Schrijver et al. 2008; DeRosa et al. 2009)
  - energy estimates do not agree
  - current-field iteration solutions not self-consistent
    - P and N choices for BCs produce different solutions

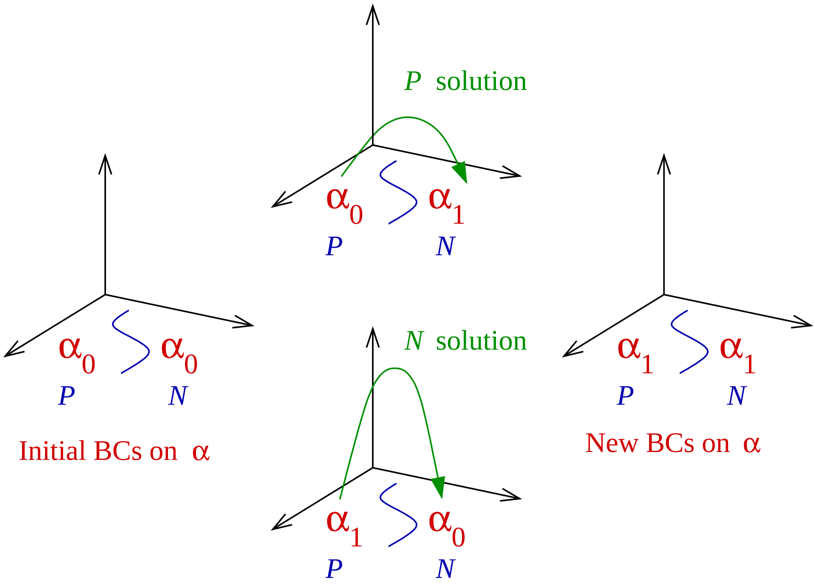
- AR 10953 on 30 April 2007 (DeRosa et al. 2009)
  - P solution (blue) and N solution (red)



- Boundary conditions inconsistent with force-free model
  - errors in field determination
  - field at photospheric level is forced (Metcalf et al. 1995)
- Necessary conditions for a force-free field (Molodenskii 1969)
  - integrals representing net force, torque
  - non-zero for solar data
  - "preprocessing" applied to enforce these conditions... (Wiegelmann et al. 2006)
    - ...but they are necessary, not sufficient
    - preprocessed BCs still inconsistent with the force-free model (DeRosa et al. 2009)
- Alternative approach:
  - find the "closest" force-free solution to the observed data

Method: Using both solutions

- Vector magnetogram provides boundary values $\alpha_0 \pm \sigma_0$
- Apply current-field iteration using α_0 over P
  - solution field lines map α_0 values in P to points in N
  - defines new boundary values $(\alpha_1 \pm \sigma_1)$ at points in N
- Apply current-field iteration using α_0 over N
  - solution field lines map α_0 values in N to points in P
  - defines new boundary values $(\alpha_1 \pm \sigma_1)$ at points in P
- The two solutions define a complete set of $\alpha_1 \pm \sigma_1$ values
  - new α boundary values at points in P and N
- We have two possible sets of boundary values
  - need to decide on a most probable value at each point



P solution
α0
α1
P
N
α0
α0
P
N
Initial BCs on α
N solution
α1
α1
P
N
New BCs on α
α1
α0
P
N

Method: Bayesian decision making

- Bayes's theorem:

$$\mathcal{P}(M|D,I) \propto \mathcal{P}(D|M,I)\mathcal{P}(M,I) \tag{5}$$

  - $\mathcal{P}$ is probability, M a model, D data, I other information
  - here M is the value of α, $D = (\alpha_1, \sigma_1)$ and $I = (\alpha_0, \sigma_0)$

- Assuming Gaussian errors

$$\begin{aligned} \mathcal{P}(D|M,I) &\propto \mathrm{e}^{-\frac{1}{2}(\alpha-\alpha_1)^2/\sigma_1^2} \\ \mathcal{P}(M,I) &\propto \mathrm{e}^{-\frac{1}{2}(\alpha-\alpha_0)^2/\sigma_0^2} \end{aligned} \tag{6}$$

- Hence

$$\mathcal{P}(M|D,I) = \exp\left[-\frac{(\alpha-\alpha_0)^2}{2\sigma_0^2} - \frac{(\alpha-\alpha_1)^2}{2\sigma_1^2}\right] \tag{7}$$

- $d\mathcal{P}/d\alpha = 0$ implies most probable value

$$\alpha = \alpha_2 = \frac{\alpha_0/\sigma_0^2 + \alpha_1/\sigma_1^2}{1/\sigma_0^2 + 1/\sigma_1^2} \tag{8}$$

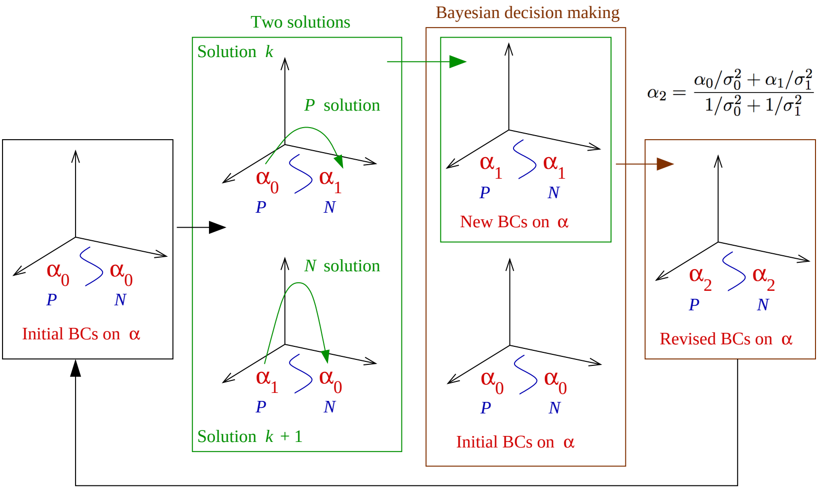
  - for constant uncertainties $\alpha_2 = \frac{1}{2}(\alpha_0 + \alpha_1)$
- Corresponding uncertainty

$$\sigma_2 = \left(\frac{1}{\sigma_0^2} + \frac{1}{\sigma_1^2}\right)^{-1/2} \tag{9}$$

  - assuming Gaussian behaviour of $\mathcal{P}$ around $\alpha = \alpha_2$
- Values α_2 still inconsistent with the force-free model
  - but closer to consistency

Method: Iteration

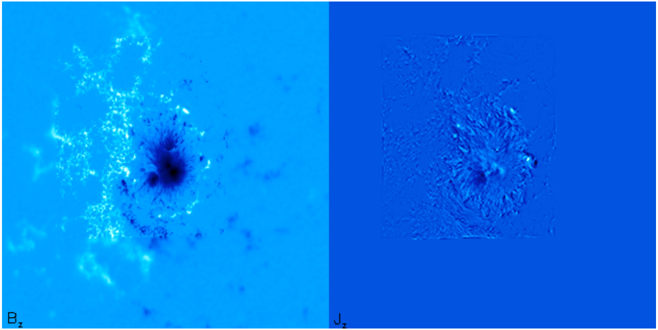
- Repeat the procedure, starting with $\alpha_2 \pm \sigma_2$
  - "self-consistency cycles"
  - label cycles by solutions ($k = 1, 2$ constructed at cycle 1)



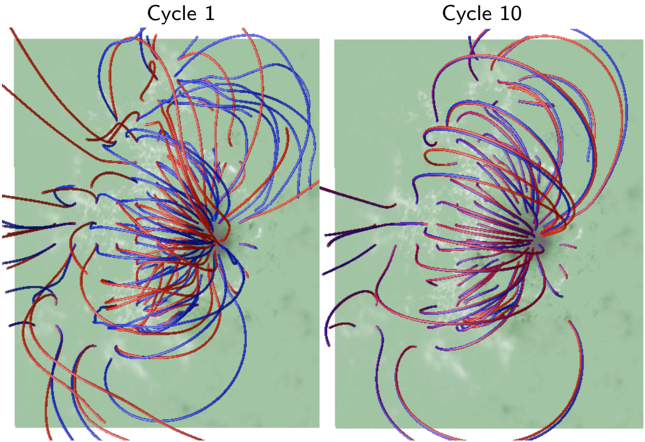
Application to Hinode/SOT data: AR 10953

- AR 10953 at 22:30 UT on 30 June 2007 (DeRosa et al. 2009)
  - (B_x, B_y, B_z) on 320×320 grid over $185.6\,(\mathrm{Mm})^2$
  - Hinode data over only part: rest has SOHO/MDI B_z values
  - α_0 obtained by centred differencing of B_x and B_y
  - $\alpha_0 = 0$ for MDI data
  - uncertainties not available, so assumed constant

Field | Current density

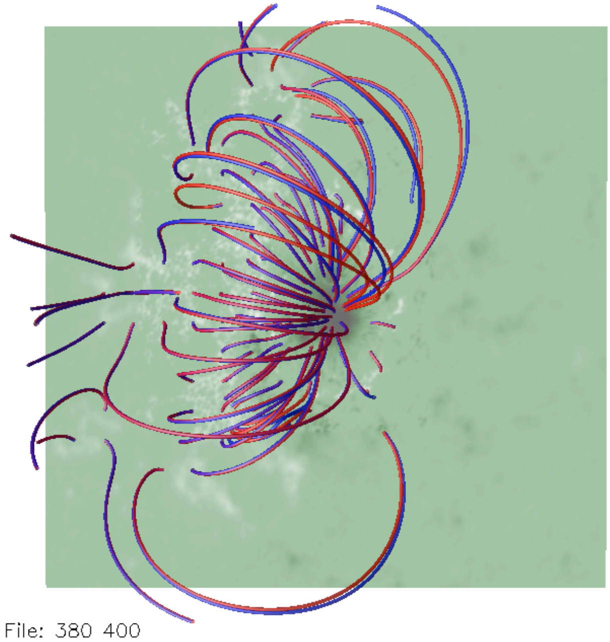


- Calculation on $320 \times 320 \times 256$ grid
  - 10 self-consistency cycles
  - 20 current-field iterations for each solution
  - run on one node (8 cores) of Science Faculty HPC



$k = 1$ (blue), $k = 2$ (red) $k = 19$ (blue), $k = 20$ (red)

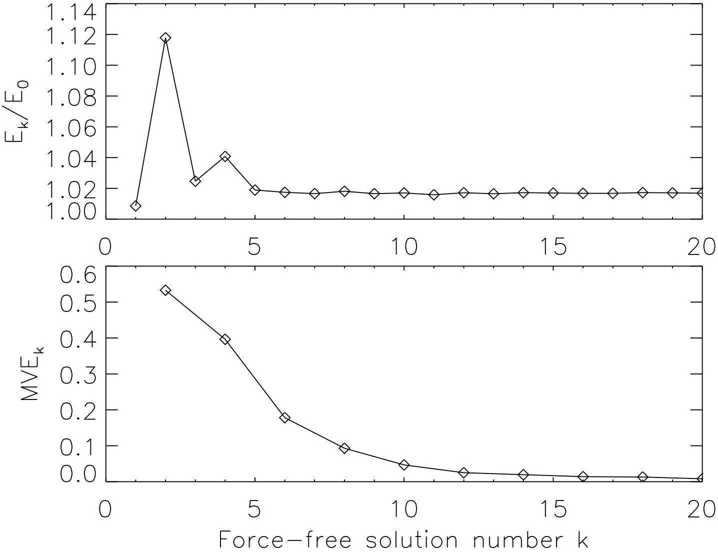
- Self-consistency achieved!



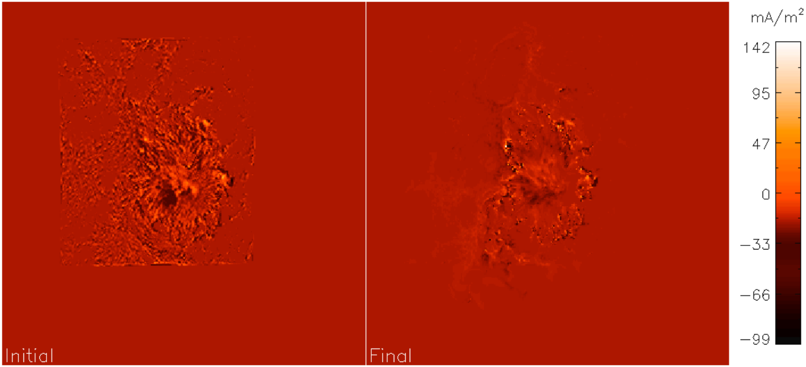
- Quantitative measures confirm convergence
- Energy E_k of solution k in units of potential energy E_0
  - E_{19} and E_{20} differ by $< 0.03\%$
- Mean vector error (Schrijver et al. 2006)

$$\mathrm{MVE}_k = \frac{1}{N_x N_y N_z} \sum_i \frac{\left|\mathbf{B}_i^{(k)} - \mathbf{B}_i^{(k-1)}\right|}{\left|\mathbf{B}_i^{(k-1)}\right|}, \qquad (10)$$

  - reduced by more than a factor of 60
- Changes in field components are large
  - RMS changes: $\Delta B_x \approx 120\,\mathrm{G}$, $\Delta B_y \approx 100\,\mathrm{G}$
  - partly due to artificial embedding within MDI data
  - cf. preprocessing: $\Delta B_x, B_y \approx 60\,\mathrm{G}$, $\Delta B_z \approx 80\,\mathrm{G}$

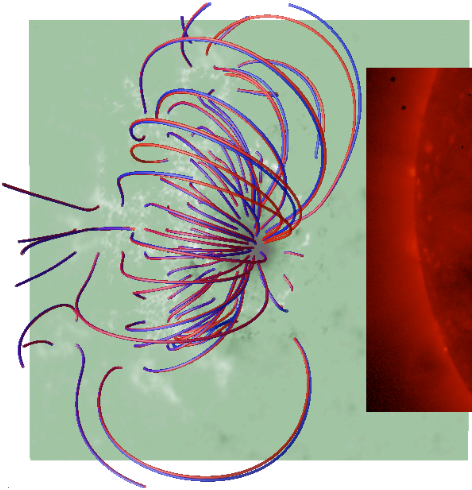

1.14
1.12
1.10
1.08
1.06
1.04
1.02
1.00
E_k/E_0
0
5
10
15
20
0.6
0.5
0.4
0.3
0.2
0.1
0.0
MVE_k
0
5
10
15
20
Force-free solution number k

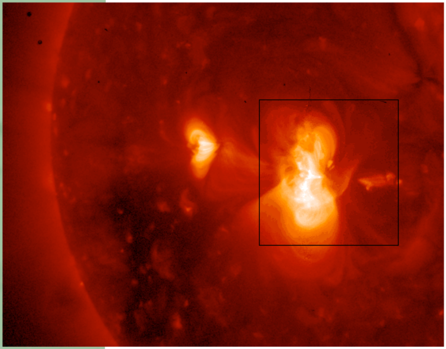
- Currents reduced in magnitude overall by averaging
  - but basic structures remain



- Application a "proof of concept"
  - uncertainties should be assigned
  - embedding in MDI data is undesirable

- Comparison with Hinode/XRT image





Summary

- Vector magnetograms enable coronal field modelling
- Nonlinear force-free model appropriate in the corona
  - but photospheric boundary data is not force-free
  - different solutions for the P and N choices for BCs on α
- Method to calculate a self-consistent nonlinear force-free field
  - calculate P and N solutions
  - use solutions and Bayes's theorem to decide on new BCs on α
  - iterate
- Demonstrated to work on Hinode/SOT data
  - a "proof of concept"
  - should be improved by including uncertainties
  - should be improved by using only vector magnetogram data